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Multisplitting and Schwarz Methods for Solving Linear Complementarity Problems†

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Abstract. In this paper we consider some synchronous and asynchronous multisplitting and Schwarz methods for solving the linear complementarity problems. We establish some convergence theorems of the methods by using the concept of M-splitting.

Key words: Linear complementarity problem; nonstationary; asynchronous; M-splitting.

AMS subject classifications: 65Y05, 65H10, 65N55

1 Introduction

Many science and engineering problems are usually induced as linear complementarity problems (LCP): finding an $x \in R^n$ such that

$$x \geq 0, \quad Ax - f \geq 0, \quad x^\top (Ax - f) = 0, \tag{1}$$

where $A \in R^{n\times n}$ is a given matrix, and $f \in R^n$ is a vector. It's necessary to establish an efficient algorithm to solving the complementarity problem (CP). Numerical methods for complementarity problems fall in two major kinds, direct method and iterative method. There have been lots of works on the solution of the linear complementarity problem [9, 10, 13–15, 18], which presented feasible and essential techniques for LCP.

Recently, Bai [1–6] presented a class of parallel iterative methods for the large sparse LCP by applying multisplitting technique. These methods are suitable to the SIMD systems or the MIMD systems, and efficient for solving the complementarity problems. One of the advantages of synchronous parallel methods is that each subdomain can be handled by a different processor of a parallel computer, which results in high computation efficiency. But, at the synchronization points, the synchronous methods need to wait the fresh information from the other processors. In this paper we consider an asynchronous parallel method, i.e., method in which the subproblem in each processor is solved anew with whatever information is available at the moment, without

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waiting for new information from all other processors. The asynchronous version converges faster than the synchronous one.

A multisplitting and additive Schwarz method for linear system:

$$Ax = b$$

is constructed by Frommer [11]. One of the advantages of this kind of parallel method is that we only have to compute some components according to the block property [11]. The numerical experiments show a slight preference for algebraic additive Schwarz methods over multisplitting methods.

In [7,8,16,17], some nonstationary multisplitting methods for linear equations are discussed. The main idea of this kind of methods is that at the lth iteration each processor j solves the system some times, using in each time the new calculated vector to update the right-hand side. The numerical results show that the nonstationary multisplitting methods are better than the standard ones.

The purpose of this paper is also on establishing efficient parallel iterative methods for solving the LCP. By skillfully using the matrix multisplitting methodology and the block property, we propose a class of stationary and nonstationary multisplitting and Schwarz methods, for solving the linear complementarity problems.

The paper is organized as follows. In Sections 2 and 3 we propose synchronous stationary and nonstationary parallel multisplitting and Schwarz methods for solving LCP and establish its convergence theorem. In Section 4 we give an asynchronous parallel multisplitting and Schwarz method for solving LCP and analysis the convergence of the algorithm.

2 Synchronous multi-splitting and Schwarz method

Machida [13] extended the multisplitting methods to the symmetric LCP, and Bai [1–4] developed a class of synchronous relaxed multisplitting methods for LCP, in which the system matrix is an H-matrix. A multisplitting and Schwarz iteration method for the system of linear equations is considered in [11]. In this section, by using multisplitting and block property techniques, we present a multisplitting and Schwarz method for the LCP (1), in which A is an M-matrix.

At first we briefly describe the notations. In R^n and $R^{n\times n}$ the relation $\geq$ denotes the natural components partial ordering. In addition, for $x, y \in R^n$ we write $x > y$ if $x_i > y_i$, $i = 1, \ldots, n$. A nonsingular matrix $A = (a_{ij}) \in R^{n\times n}$ is termed M-matrix, if $a_{ij} \leq 0$ for $i \neq j$ and $A^{-1} \geq 0$.

Definition 2.1. A splitting $A = M - N$ is termed M-splitting of matrix A if M is an M-matrix and $N \geq 0$.

Definition 2.2. ([11]) Let $A \in R^{n\times n}$ be nonsingular. A collection of m splittings $A = M_l - N_l$, $l = 1, \ldots, m$, and m^2 nonnegative diagonal matrices $E_{l,i} \in R^{n\times n}$ such that $\sum_{i=1}^{m} E_{l,i} = I$ for $l = 1, \ldots, m$ is called a weighted additive Schwarz-type splitting of A.

Lemma 2.1. ([11]) *Let A be an M-matrix, then the M-splitting $A = M - N$ of the matrix A is convergent.*

Next we consider the following multisplitting method.

Algorithm 1. (Synchronous multisplitting and Schwarz method)

1) Give an initial vector $x^{0,l} \in R^n$, $l = 1, \ldots, m$ and $k = 0$.