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The Dual Functionals for the Generalized Ball Basis of Wang-Said Type and Basis Transformation Formulas†

Ping Jiang[1,*], Hongyi Wu[2] and Jieqing Tan[1]

[1] *Department of Mathematics, School of Sciences/ School of Computer and Information, Hefei University of Technology, Hefei 230009, China.*
[2] *Department of Mathematics, School of Sciences, Hefei University of Technology, Hefei 230009, China.*

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Abstract. The generalized Ball curves of Wang-Said type with a position parameter L not only unify the Wang-Ball curves and the Said-Ball curves, but also include several useful intermediate curves. This paper presents the dual functionals for the generalized Ball basis of Wang-Said type. The relevant basis transformation formulae are also worked out.

Key words: Generalized Ball basis of Wang-Said type; dual functionals; Bernstein basis; basis transformation.

AMS subject classifications: 65D17

1 Introduction

Over the past several decades, there have been quite a few studies on generalized Ball curves. The cubic Ball curves were first introduced by Ball (1974) in a lofting surface program CONSURF at the British Aircraft Corporation [1, 2], then they were extended to arbitrary high degrees [12] by Wang (1987) and arbitrary odd degrees [11] by Said (1989), respectively. The former are called Wang-Ball curves as named in [7] while the latter are called Said-Ball curves or generalized Ball curves. Recently Wu presented two families of new generalized Ball curves [13,14]: the generalized Ball curves of Said- Bézier type, and the other is generalized Ball curves of Wang-Said type (WSGB curves). Several researchers [3-6, 8-10, 15] have theoretically come to the conclusion that generalized Ball curves are more efficient than Bézier curves in calculations, the degree elevation and reduction. However, no detailed discussions about WSGB curves have ever been reported, which clearly results in negative influence on the applications of the WSGB curves in the field of CAGD.

*Correspondence to: Ping Jiang, Department of Mathematics, School of Sciences/ School of Computer & Information, Hefei University of Technology, Hefei 230009, China. Email: jiangpin_72@sina.com
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The emphasis of this paper is on the exploration of the dual functionals for WSGB basis, and efforts are also made to investigate the basis transformation formula between Bernstein basis and WSGB basis.

2 Generalized ball curves of Wang-Said type

For any integer $n \geq 2$, let $m = \lfloor n/2 \rfloor$. The generalized Ball curves of Wang-Said type (WSGB curves) with position parameter $L(L = 0, 1, \cdots, m-1)$ are defined by

$$\boldsymbol{r}(u; n, L) = \sum_{i=0}^{n} \boldsymbol{p}_i \beta_i(u; n, L), \qquad 0 \leq u \leq 1,$$

where $\boldsymbol{p}_i (i = 0, 1, \cdots, n)$ are the control points, and $\beta_i(u; n, L)(i = 0, 1, \cdots, n)$ are the WSGB basis functions,

$$\beta_i(u; n, L) = \begin{cases} \binom{\lfloor n/2 \rfloor - L + i}{i} u^i v^{\lfloor n/2 \rfloor - L + 1}, & 0 \leq i \leq \lfloor n/2 \rfloor - L - 1 \\ 2^{i - \lfloor n/2 \rfloor + L} \binom{2\lfloor n/2 \rfloor - 2L}{\lfloor n/2 \rfloor - L} u^i v^{i+2}, & \lfloor n/2 \rfloor - L \leq i \leq \lfloor n/2 \rfloor - 1 \\ 2^L \binom{2\lfloor n/2 \rfloor - 2L}{\lfloor n/2 \rfloor - L} u^{\lfloor n/2 \rfloor} v^{\lceil n/2 \rceil}, & i = \lfloor n/2 \rfloor \\ 2^L \binom{2\lfloor n/2 \rfloor - 2L}{\lfloor n/2 \rfloor - L} v^{\lfloor n/2 \rfloor} u^{\lceil n/2 \rceil}, & i = \lceil n/2 \rceil \\ \beta_{n-i}(1-u; n, L), & \lceil n/2 \rceil + 1 \leq i \leq n \end{cases}, \qquad (1)$$

where $v = 1 - u$, $\lfloor x \rfloor$ and $\lceil x \rceil$ denote the greatest integer less than or equal to x and the least integer greater than or equal to x, respectively. Obviously, WSGB curves unify the Said-Ball curves ($L = 0$) and Wang- Ball curves ($L = m - 1$) , and include some intermediate curves ($L = 1, 2, \cdots, m - 2$).

For any real number α and integer m, k, we define

$$\binom{\alpha}{k} = \frac{\alpha(\alpha - 1) \cdots (\alpha - k + 1)}{k!},$$

and use the convention

$$\binom{\alpha}{0} = 1; \qquad \binom{\alpha}{k} = 0, \quad \text{if} \;\; k < 0.$$

It is easy to see that the following equality holds.

$$(-1)^k \binom{m + k - 1}{k} = \binom{-m}{k}.$$

In the sequel we will also make use of the following combinatorial identities [16]

$$\sum_{k=0}^{n} \binom{x+k}{k} \binom{y+n-k}{n-k} = \binom{x+y+n+1}{n},$$

$$\sum_{k=0}^{n} (-1)^k \binom{x+k}{r} \binom{n}{k} = (-1)^n \binom{x}{r-n},$$

where n, k are positive integers, and x, y, r are real numbers.