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A Filter-Based Pattern Search Method for Unconstrained Optimization

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Abstract. We discuss a filter-based pattern search method for unconstrained optimization in this paper. For the purpose to broaden the search range we use both filter technique and frames, which are fragments of grids, to provide a new criterion of iterate acceptance. The convergence can be ensured under some conditions. The numerical result shows that this method is practical and efficient.

Key words: Direct search; pattern search; positive base; filter; derivative-free optimization.

AMS subject classifications: 65K, 90C

1 Introduction

With the rising of evolutionary computation, direct search methods for unconstrained optimization are emergently needed and are consequently developing rapidly.

Direct search methods are a kind of derivative-free methods. They are designed for the functions whose information of the derivatives are unavailable or unreliable. Direct search methods have a lot of kinds of techniques, such as line search methods; pattern search methods; simplex methods; conjugate direction methods; linear approximation methods; quadratic approximation methods and so on. Among them pattern search methods gain a lot of interests from many scholars [3, 9, 11, 12].

Pattern search methods were first proposed by Box [1] in the 1950s and Hooke & Jeeves [8] in the early 1960s. They developed rapidly in the next ten years. However, it follows from the lack of proofs of convergence and slow rate of convergence in some cases that pattern search methods were rarely used then. Nowadays, because of the appearance of mathematical analysis, pattern search methods are reemphasized by many scholars. Generally speaking, pattern search methods generate a sequence of iterates $\{x^{(k)}\}$ without using any information of the derivatives, including gradient and second-order derivative , of the objective function. They only depend on function values. In fact, ordinal information about function values is enough. At each iteration, the objective function is evaluated at a finite number of trial points. And the purpose is to look for one which can yield a lower function value than the current iterate. If such a point is

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found, set it to be the new iterate and the iteration is called successful. Otherwise, we declare it unsuccessful and update trial points.

Now we use a matrix $D^{(k)} \in \mathbb{R}^{n\times p^{(k)}}$ to indicate the set of search directions on the kth iteration, where $p^{(k)}$ is the number of directions in the set. $\Delta^{(k)}$ is a rational scale factor. Then the generalized pattern search process is outlined as follows:

Let initial iterate $x^{(0)} \in \mathbb{R}^n$, $D^{(0)} \in \mathbb{R}^{n\times p^{(0)}}$, $\Delta^{(0)} > 0$, $k = 0$.

While (Stopping conditions do not hold) do

Step 1. Find a step $s_i^{(k)} \in \Delta^{(k)} D^{(k)}, i \in \{1, 2, \cdots, p^{(k)}\}$.

Step 2. If $f(x^{(k)} + s_i^{(k)}) < f(x^{(k)})$, then $x^{(k+1)} = x^{(k)} + s_i^{(k)}$. Otherwise, $x^{(k+1)} = x^{(k)}$.

Step 3. Update $D^{(k)}$ and $\Delta^{(k)}$to $D^{(k+1)}$ and $\Delta^{(k+1)}$, $k = k + 1$.

Pattern search methods can remain until now because of their simplicity and practical use. More recent works in unconstrained pattern search algorithms include the multi-directional search of Dennis and Torczon [6], and Torczon's work on generalized pattern search methods [13]. In addition, other similar work is due to Coope and Price on the grid search framework [3, 4]. Herein we propose a new method in which we look for a new iterate by filter approach proposed by Fletcher and Leyffer [7] to be applied to numerical optimization as a way to acceleraten convergence.

We consider the unconstrained optimization ploblem

$$\min f(x) \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$. Here $f(x)$ is assumed to be a continuously differentiable function whose information of derivatives are unreliable or unavailable. Each algorithm conforming to the framework we suggest generates a sequence of iterates $\{x^{(k)}\}_{k=1}^{\infty}$. At each iteration, we will calculate the values of $f(x)$ and filter function at several points around $x^{(k)}$ including points on the frame generated from $x^{(k)}$. The next iterate will be chosen by our new criterion which is much looser than before. So the search range will be wider and wider, even full of the whole space. Although the function values $\{f(x^{(k)})\}_{k=1}^{\infty}$ are not surely monotonically decreasing, a subsequence of the iterates can converge to one or more stationary points of $f(x)$ under some conditions.

Some symbols used in this paper are introduced as follows:

λ:	mesh size.
V:	a base in $\mathbb{R}^n$.
V_+:	an ordered positive base generated from V.
υ_i:	the member in V or V_+.
$\|A\|$:	the number of volumns in matrix A.
M:	a filter.
Superscript k:	the kth iteration.

We will discuss our framework in Section 2. The convergence of algorithms conforming to our framework is proved in Section 3. Finally, we will show the numerical report.

2 Framework

The algorithms in general pattern seach methods choose any lewer point along the directions in the set of search directions to be the new iterate. When no improved point can be found at current iterate, the grid will be finer and a new search begins. Meanwhile we will call the current iterate pattern search point. However, in fact the simple decrease[9] will increase the number of iterations. And if the minimizer of $f(x)$ is very far from the initial iterate, it will take long time to arrive at the minimizer because the initial stepsize will be reduced gradually in the