J. Partial Differential Equations
Vol. 2, *No.* 3(1989), 30—38

A NOTE ON THE GENERALIZED CARLEMAN BOUNDARY VALUE PROBLEM FOR THE SYSTEM WITH CONSTANT COEFFICIENTS OF THE CLASS E_2

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(Received July 23, 1987; revised December 1, 1988)

Abstract

The main purpose of this note is to consider the generalized Carleman boundary value problem for the elliptic equations with constant coefficients of the second type in the simple connected domain of complex plane. Making use of Fredholm integral equations and the classical theory of the boundary value problem for analytic functions, we obtain the existence, uniqueness and the representation of solution for above Carleman boundary value problem.

Some boundary value problems of complex equations with variable coefficients of second order have been studied widely while all the coefficients of equation are sufficiently small.

Certain boundary value problems for the system with constant coefficients of the class E_2

$$\frac{\partial^2}{\partial \bar{z}^2}[(\lambda-k)(k+1)w+(\lambda+k)(k-1)\overline{w}]$$

$$-\frac{\partial^2}{\partial \bar{z}\partial z}[(\lambda-k)(k-1)w+(\lambda+k)(k+1)\overline{w}]=0 \qquad (1)$$

$0<k<1, \lambda\neq 0, 1, k^2$, in unit disc, were studied by Hua Luogeng and others [1], and the existence and the representation of solutions of these boundary value problems were established by them.

One sees that under conformal mapping $\zeta=\omega(z)$ converting the simple connected domain into unit disc, equation (1) has been transformed into an equation with variable coefficients in respect of ζ. In addition, both the equation with constant coefficients and the equation under conformal mapping with variable coefficients, in general, all the coefficients of the equation, are not always the case such that they satisfy sufficiently small condition.

It may be seen that even if the equation (1) with constant coefficients in respect of the boundary value problem for general domain may be also discussed now.

In addition, as we know from reference [2] that the displacement equations of the plane elasticity for orthogonal anisotropic or homogeneous media are just Hua's equation (1), this is what we call the mechanical explanations of Hua's equation (1). It shows that the research of boundary value problem for equation (1), such as Dirichlet problem, Carleman problem, has important applied meaning too.

In this note we consider the generalized Carleman boundary value problem for equation (1), that is, find out the solution $w(z)\in C^2(D^+)\bigcap C^1(\overline{D^+})$ of equation (1) in domain D^+ which fulfils the boundary condition on boundary curve Γ

$$w^+[\alpha(t)] = a(\lambda,k)w^+(t) + b(\lambda,k)\overline{w^+(t)} + c(\lambda,k) \tag{2}$$

Domain D^+ is a finitely simple connected domain bounded by Γ satisfying Liapunov condition. Function $\alpha(z)$ is a homeomorphism mapping domain $\overline{D^+}$ onto itself, which satisfies conditions as following:

$$\alpha[\alpha(z)] \equiv z \ , \ z \in D^+$$

$$\alpha'(t) \in H_\mu(\Gamma) \ , \ 0 < \mu < 1$$

and $\alpha(z)$ is a positively oriented displacement $\alpha_+(t)$ or a negatively oriented displacement $\alpha_-(t)$ on Γ. As for $a(\lambda,k)$, $b(\lambda,k)$ and $c(\lambda,k)$, they are real constants depending on given constants λ and k.

In order to rule out the hyperdefinite property for boundary value problem, therefore the boundary condition (2) may be reduced to one of the Carleman or Carleman type boundary value problems as follows

$$w^+[\alpha(t)] = -w^+(t) + c(\lambda,k) \tag{3}$$

$$w^+[\alpha(t)] = \pm w^+(t) \tag{4}$$

$$w^+[\alpha(t)] = -\overline{w^+(t)} + c(\lambda,k) \tag{5}$$

$$w^+[\alpha(t)] = \pm \overline{w^+(t)} \tag{6}$$

As we know that boundary value problem (3) or (4) is solvable if $\alpha(t)\equiv\alpha_-(t)$; boundary value problem (5) or (6) is solvable if $\alpha(t)\equiv\alpha_+(t)$. Consequently we shall restrict ourselves to these solvable cases in this note.

We state the following

Lemma 1 *Suppose that $c(\lambda,k)\neq 0$, then the BVP*

$$w^+[\alpha_-(t)] = -w^+(t) + c(\lambda,k) \tag{3}$$

for the equation (1) in domain D^+ possesses a nontrivial solution $w(z)$, which has the form