J. Partial Differential Equations
Vol. 3, *No.* 2(1990), 77—84

HIGHER-ORDER NONLINEAR SYSTEM OF EQUATIONS OF CHANGING TYPE

Sun Hesheng
(Centre for Nonlinear Studies,
Institute of Applied Physics and Computational Mathematics, Beijing)

(Received August 10, 1988; revised February 17, 1989)

Abstract In practical problems there appears higher-order equations of changing type([1]). But, there are only a few of papers concerning this type of equations([2]—[6]).

In this paper, the global existence of regular solutions to the initial-boundary value problem for a class of higher-order system of equations of changing type with a strong nonlinear term is studied.

Key Words Equations of changing type; higher-order nonlinear system; global existence.

Classification 35M05

In domain $Q_T=\{(x,t)\,|\,0\leqslant x\leqslant l, 0\leqslant t\leqslant T\}$ we consider the nonlinear system of equations of changing type

$$(K(t)u_t)_t + (-1)^{M-1}Au_{x^{2M}} - \mathrm{grad}F(u) = f(x,t) \tag{1}$$

and initial-boundary value problem

$$\begin{cases} u_{x^k}(0,t) = u_{x^k}(l,t) = 0, \quad k = 0,1,\cdots M-1, t\in[0,T] \\ u(x,0) = \varphi(x), \quad x\in[0,l], \varphi^{(k)}(0) = \varphi^{(k)}(l) = 0, k = 0,1,\cdots,M-1 \end{cases} \tag{2}$$

where $M\geqslant 1$ is an integer; u, f and φ are N-dimensional vectors: $=(u_1,\cdots,u_N)$, $f=(f_1,\cdots,f_N)$, $\varphi=(\varphi_1,\cdots,\varphi_N)$; $K(t)$ is a $N\times N$ diagonal matrix; $K(t)=\mathrm{diag}\{k_1(t),\cdots,k_N(t)\}$; A is a $N\times N$ constant matrix; F is a nonlinear function of vector u.

Assume that the coefficients and functions in (1) and (2) satisfy the following conditions:

$$\begin{cases} \text{(i)} & k_i(t) > 0 \text{ when } t = 0; k_i(t) \geqslant 0 \text{ when } t \in (0,t_0) \\ & k_i(t) = 0 \text{ when } t = t_0; k_i(t) < 0 \text{ when } t \in (t_0,T] \\ & k_i(t) \in C^2[0,T], k_i'(t) \leqslant -k_0 < 0, \forall t \in [0,t_0], i = 1,\cdots,N. \\ \text{(ii)} & A \text{ is a symmetric positively definite matrix:} \\ & (A\xi,\xi) \geqslant a_0|\xi|^2, \forall \xi = (\xi_1,\cdots,\xi_N) \in R^N, a_0 > 0 \\ \text{(iii)} & F(u) \geqslant 0, F \in C^2, F(\varphi(x)) \in L_1(0,l) \text{ and } F \text{ satisfies} \\ & \left|\dfrac{\partial^2 F}{\partial u_i \partial u_j}\right| \leqslant C_1|u|^\rho + C_2, \quad i,j = 1,\cdots,N \\ & \text{where } \rho \text{ is an arbitrary non-negative real number.} \\ \text{(iv)} & \varphi_i \in H^{2M}(0,l), f_i \in H^1(Q_T), i = 1,\cdots,N. \end{cases} \tag{3}$$

It is evident that, in the case $M=1$, the system (1) is a second order system of elliptic type in the domain Q_{t_0}, and is a second order system of hyperbolic type in the domain $Q_T \backslash \overline{Q}_{t_0}$; $t=t_0$ is its degenerate line, hence (1) is a nonlinear system of equations of mixed type. In the case $M>1$, system (1) is of hypoelliptic type in Q_{t_0}, and is of ultrahyperbolic type in $Q_T \backslash \overline{Q}_{t_0}$, hence (1) is a nonlinear system of equations of changing type. Moreover, the system (1) belongs to the second kind of degenerate type. Hence, besides the boundary conditions for x, only one initial condition for t on the non-degenerate boundary of the elliptic domain is needed in order that the problem is well-posed. (Notice that, the degenerate line of $k_i(t)$ may be different from different i.)

Assume that on the degenerate line $t=t_0$ the following normal connected conditions are satisfied:

$$\lim_{t \to t_0 - 0} (-k_i(t) u_{ix^s t^{r+1}}, u_{ix^s t^{r+1}}) = \lim_{t \to t_0 + 0} (-k_i(t) u_{ix^s t^{r+1}}, u_{ix^s t^{r+1}}), \quad 0 \leqslant s + rM \leqslant M,$$
$$s = 0,1,\cdots,M; r = 0,1; i = 1,\cdots,N \tag{4}$$

For any two vectors u and v we define

$$(u,v)(t) = \int_0^l u \cdot v dx, \ [u,v](t) = \int_0^t (u,v)(t) dt \tag{5}$$

and define the norms:

$$|u(\cdot,t)|^2_{L_2(0,l)} = (u,u)(t), \quad \| u \|^2_{L_2(Q_t)} = [u,u](t)$$

$$|u(\cdot,t)|^p_{L_p(0,l)} = \sum_{i=1}^N |u_i(\cdot,t)|^p_{L_p(0,l)}, \quad \| u \|^p_{L_p(Q_t)} = \sum_{i=1}^N \| u_i \|^p_{L_p(Q_t)}$$

Lemma 1 *Under the conditions* (3), *any solution of problem* (1)—(2) *satisfies the estimate*

$$\| u_t \|^2_{L_2(Q_t)} + \| u_{x^M} \|^2_{L_2(Q_t)} + |u_{x^M}(\cdot,t)|^2_{L_2(0,l)}$$