J. Partial Differential Equations
Vol. 3, *No.* 2(1990), 27—40

STATIC SOLUTIONS OF MIXED BURGERS-KDV EQUATION Ⅱ [①]

Guan Keying
(Dept. of Math., Beijing Univ. of Aeronautics and Astronautics)
(Received March 3, 1988; revised July 30, 1988)

Abstract Based on the method of qualitative research in ordinary differential equations, it is proved that, for any given positive β and γ, and for any given real a, b and c, the Burgers-KdV equation

$$u_t + uu_x - \gamma u_{xx} + \beta u_{xxx} = 0$$

has at least one, but at most finite static solutions satisfying the same boundary conditions

$$u(0,t) = a, u(1,t) = b \text{ and } u_x(1,t) = c$$

on the interval $[0,1]$ of x. Some sufficient conditions on the global stability for certain static solutions are given.

Key Words Boundary conditions; number of static solutions; global stability; dissipation; dispersion.

Classification 35M05.

1. Introduction

In [1], we have shown that, for any positive β and γ, and for any real a, b and c, the Burgers-KdV equation

$$u_t + uu_x - \gamma u_{xx} + \beta u_{xxx} = 0 \tag{1.1}$$

has infinitely many static solutions satisfying the boundary conditions

$$u(0,t) = a, u_x(0,t) = b \text{ and } u(1,t) = c \tag{1.2}$$

on the interval $[0,1]$ of x, and these static solutions differ from each other mainly by their numbers of extremum points.

In this paper we shall show that, if the boundary conditions (1.2) are changed into

$$u(0,t) = a, u(1,t) = b \text{ and } u_x(1,t) = c \tag{1.3}$$

---

① This research is financed by the National Foundation of Natural Science of China.

then the equation (1. 1) has at least one, but at most finite static solutions, i. e., the change of ways in assigning the boundary conditions leads to a great change in quality of the number of the static solutions. In addition, we will also show that this change of the boundary conditions has a strong influence on the stability of the static solutions. And we will give some sufficient conditions on the global stability for certain static-solutions.

Because the Burgers-KdV equation (1. 1) is the simplest nonlinear wave equation with the mixed effect of dissipation and dispersion (see [2], [3]), the strong dependence of the number and stability of static solutions on the boundary conditions must reflect some fundamental properties of the system with the effect of dissipation and dispersion.

2. Existence and Number of Static Solutions

In [1], we have known that the static solution of equation (1. 1) satisfies the following ordinary differential equation

$$u^2/2 - \gamma u' + \beta u'' = k \tag{2. 1}$$

where $k \in R$ is an integral constant, and $u' = du/dx$. And under the transformation

$$X = u(x), Y = u'(x)$$

(2. 1) is changed into the following autonomous system

$$\begin{cases} X' = Y \\ Y' = \dfrac{k}{\beta} + \dfrac{\gamma}{\beta} Y - \dfrac{1}{2\beta} X^2 \end{cases} \tag{2. 2}$$

For given β, γ, a, b, c and k, there exists a unique integral curve passing through the point $P_0 \equiv (b, c)$ on the (X, Y) plane when

$$x = x_0 \equiv 1$$

The part of this integral curve corresponding to $x \leqslant 1$ is denoted by $L(k)$, and the moving point on $L(k)$ is denoted by

$$P(x,k) \equiv (X(x,k), Y(x,k)), \quad x \in (-\infty, 1] \tag{2. 3}$$

Obviously

$$P_0 = P(1,k) = (X(1,k), Y(1,k)) = (b,c)$$

Generally, for any given $k \in R$, the point

$$P(0,k) = (X(0,k), Y(0,k))$$