J. Partial Differential Equations
Vol. 5, *No.* 2 (1992), 81–93

ON THE INITIAL BOUNDARY VALUE PROBLEMS FOR TWO DIMENSIONAL SYSTEMS OF ZAKHAROV EQUATIONS AND OF COMPLEX-SCHRÖDINGER–REAL-BOUSSINESQ EQUATIONS [1)]

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(Received July 14, 1988; revised May 15, 1991)

Abstract In this paper the author considers the initial boundary value problems for the system of Zakharov equations (Za) and for the system of complex-Schrödinger–real-Boussinesq equations (SBq) in a two-dimensional domain. He proves the global existence and uniqueness of smooth solutions of (SBq) and of (Za) by means of continuous extension method and *a priori* estimates.

Key Words (Za) equations (SBq) equations; compatibility conditions; *a priori* estimate.

Classification 35Q20.

1. Introduction

In this paper we consider the initial boundary value problems for two-dimensional system of Zakharov equations (Za)

$$\mathrm{i}\ \vec{\varepsilon}_t + \Delta\vec{\varepsilon} - u\vec{\varepsilon} = \vec{0} \qquad (1.1)$$

$$t \geq 0,\ x \in \Omega$$

$$u_{tt} - \Delta u = \Delta|\vec{\varepsilon}|^2, \qquad (1.2)$$

$$\vec{\varepsilon}(0,x) = \vec{\varepsilon}_0(x),\ u(0,x) = u_0(x),\ u_t(0,x) = u_1(x),\ x \in \Omega$$

$$\vec{\varepsilon}(t,x) = \vec{0}, u(t,x) = 0,\ t \geq 0,\ x \in \partial\Omega \qquad (1.3)$$

and for system of complex-Schrödinger–real-Boussinesq equations (SBq) which arose in plasma physics (see [11], [12], [15], [17])

$$\mathrm{i}\ \vec{\varepsilon}_t + \Delta\vec{\varepsilon} - u\vec{\varepsilon} = \vec{0} \qquad (1.4)$$

$$t \geq 0,\ x \in \Omega$$

$$u_{tt} - \Delta u + \alpha^2\Delta^2 u - \Delta F(u) = \Delta|\vec{\varepsilon}|^2, \qquad (1.5)$$

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1) The project supported by the Special Fund of Scientific Research of Docterial Subject Station of High Education

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$$\vec{\varepsilon}(0,x)=\vec{\varepsilon}_0(x),\ u(0,x)=u_0(x),\ u_t(0,x)=u_1(x),\ x\in\Omega$$
$$\vec{\varepsilon}(t,x)=\vec{0},u(t,x)=\alpha\Delta u(t,x)=0,\ t\geq 0,\ x\in\partial\Omega \tag{1.6}$$

where $\Omega\subset \boldsymbol{R}^2$ is a bounded or exterior domain with a closed and Lipschitz continuous curve $\partial\Omega$ as its boundary, $x=(x_1,x_2)\in \boldsymbol{R}^2$, $\Delta=\dfrac{\partial^2}{\partial x_1^2}+\dfrac{\partial^2}{\partial x_2^2}$, $\vec{\varepsilon}(t,x)=(\varepsilon^1(t,x),\varepsilon^2(t,x))$ and $\vec{\varepsilon}_0(x)=(\varepsilon_0^1(x),\varepsilon_0^2(x))$ are complex vector-valued functions; $u(t,x)$, $u_0(x)$, $u_1(x)$ and $F(z)$ ($z\in \boldsymbol{R}$) are real-valued functions. F is sufficiently smooth, $F(0)=0$. Constant $\alpha>0$. If $F=0$, $\alpha=0$, then the problem (1.4)–(1.6) is just the same with (1.1)–(1.3).

In Section 2 we use iteration method to prove the local existence of solution for the problem (1.4)–(1.6). In Section 3 we establish *a priori* estimates for the solution of (1.4)–(1.6) so that we can obtain the global existence of solution. To the problem (1.1)–(1.3) we can not directly apply Galerkin method as done in [5] [6] in the case that Ω is unbounded. However, we can establish *a priori* estimates that are independent of α for the solutions of (1.4)–(1.6) when $F=0$ and $0<\alpha\leq 1$. As $\alpha\to 0$, these solutions converge in some appropriate spaces, and then the limit is the solution of the problem (1.1)–(1.3).

In the sequent sections, all Sobolev spaces in discussion are complex-valued and their notations are the same as in [1]. For a complex number z, $\bar{z}$ is its conjugate. $\|\cdot\|_p$ is the norm of space $L^p(\Omega)(1\leq p\leq\infty)$, especially $\|\cdot\|=\|\cdot\|_2$. For $l\geq 0$, $\partial_x^l=\{\partial_{x_1}^{l_1}\partial_{x_2}^{l_2};\ l_1+l_2=l\}$. Denote $D=\{u\in H^4(\Omega);\ u$ and Δu are in $H_0^1(\Omega)\}$, which is a closed subspace of $H^4(\Omega)$. For a function $v\in\bigcap_{j=0}^{[\frac{k}{2}]}C^j([0,T],\ H^{k-2j}(\Omega))$, $|||v(t)|||_k=\sum_{j=0}^{[\frac{k}{2}]}\|v(t)\|_{H^{k-2j}(\Omega)}$. That a vector belongs to some space means all its components are in this space. C is a general constant, which is independent of α, $C(\alpha)$ is a constant depending on α. Both C and $C(\alpha)$ may assume different values in different cases.

2. The Local Existence of the Problem (1.4)–(1.6)

To solve the problem (1.4)–(1.6) we need some preliminary existence and energy inequalities of solutions for the following initial boundary value problems of linear equations. The first one is

$$u_{tt}-\Delta u+\alpha^2\Delta^2 u=f(t,x),\ \ t\geq 0,\ x\in\Omega \tag{2.1}$$

$$u(0,x)=u_0(x),u_t(0,x)=u_1(x),\ x\in\Omega$$
$$u(t,x)=\alpha\Delta u(t,x)=0,\ \ t\geq 0,\ x\in\partial\Omega \tag{2.2}$$