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BOUNDARY VALUE PROBLEMS FOR COMPOSITE TYPE SYSTEMS OF SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS

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Abstract Boundary value problems for composite systems of partial differential equations are investigated in this paper. The uniqueness and existence of classical solutions and generalized solutions are studied.

Key Words Composite system; boundary value problem; Fourier method; classical solution; generalized solution
Classifications 35M05, 35C10

1. Introduction

The partial differential equations of composite type was studied for the first time by J. Hadamard [1] in 1933, the equation concerned was

$$\frac{\partial}{\partial x}\Delta u = 0 \tag{1.1}$$

it was deduced from problems in fluid dynamics. In recent years, there are more and more research papers on composite type equations and systems, for example, T. D. Dzuraev's work [2] on third and fourth order equations, among the equations considered was the following one

$$\frac{\partial^2}{\partial x^2}\Delta u = 0 \tag{1.2}$$

In China, Hua Lookeng, Lin Wei and Wu Ciqian began their work from 1960's on second order systems of partial differential equations [3]

$$\left[A\frac{\partial^2}{\partial x^2} + 2B\frac{\partial^2}{\partial x \partial y} + C\frac{\partial^2}{\partial y^2}\right]\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.3}$$

where A, B and C are 2×2 real constant matrices. They have studied two kinds of systems of composite type, the first kind is the one whose biquadratic characteristic equation

$$|A\xi^2 + 2B\xi + C| = 0$$

has a pair of complex roots and a double real root, it was reduced into the following canonical form [3, p145]

$$L_0\begin{pmatrix} u \\ v \end{pmatrix} = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\frac{\partial^2}{\partial x^2} + 2\begin{pmatrix} 0 & 1 \\ b & 0 \end{pmatrix}\frac{\partial^2}{\partial x \partial y} + \begin{pmatrix} \lambda & 0 \\ 0 & 0 \end{pmatrix}\frac{\partial^2}{\partial y^2}\right]\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\lambda - 4b = 1, \quad \lambda \neq 1 \tag{1.4}$$

by linear transformations.

The general solutions of system (1.4) are the representations [3, p145]

$$\begin{aligned} u(x,y) &= f_1(y) + u_0(x,y) \\ v(x,y) &= -\lambda x f_1(y)/2 + f_2(y) + 2bv_0(x,y) \end{aligned} \tag{1.5}$$

where $f_1(y)$ and $f_2(y)$ are arbitrary real functions, $u_0(x,y) + iv_0(x,y)$ is an arbitrary analytic function. An interesting fact we noticed is that the solutions u are v of form (1.5) satisfy Equations (1.1) and (1.2) respectively.

In this paper, we will study boundary value problems for composite system (1.4).

2. Boundary Value Problem on a Rectangular Domain

Let the rectangular domain be $R = \{(x,y) : 0 < x < h,\ 0 < y < 1\}$, the boundary value problem to be considered in this section is to find functions $u \in C^1(\bar{R}) \cap C^2(R)$ and $v \in \{v \in C^1(\bar{R}) : |v_x \in C^1(R)\}$ such that

$$L_0\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ (x,y) \in R \tag{2.1}$$

$$\begin{cases} u|_{x=0} = \varphi_0(y), & u|_{x=h} = \varphi_1(y), \quad 0 \le y \le 1 \\ v|_{x=0} = \psi_0(y), & v|_{x=h} = \psi_1(y), \quad 0 \le y \le 1 \\ u|_{y=0} = u|_{y=1} = 0, & 0 \le x \le h \end{cases} \tag{2.2}$$

where $\varphi_k(y),\ \psi_k(y) \in C^3[0,1],\ k = 0,1$, and satisfy

$$\varphi_k(0) = \varphi_k(1) = \varphi_k''(0) = \varphi_k''(1) = \psi_k'(0) = \psi_k'(1) = 0, \ k = 0,1$$

For nonnegative integer i, denote

$$\begin{aligned} \Delta(i,\lambda) &= \lambda\pi ih(1 + \exp(-\pi ih)) + 2(1-\lambda)(1 - \exp(-\pi ih)) \\ I(\lambda) &= \{i : \Delta(i,\lambda) = 0\} \\ N(i,\lambda) &= -(\lambda\pi ih(\exp(\pi ih) - 1)^{-1} - 4b)(u_i(h) - u_i(0)) \\ &\quad + \lambda\pi ihu_i(0) + 2(v_i(h) - v_i(0)). \end{aligned}$$