J.Partial Diff. Eqs. 8(1995), 174–192
©International Academic Publishers Vol.8, No.2

ANALYTIC SOLUTIONS OF NOYES-FIELD SYSTEM FOR BELOUSOV-ZHABOTINSKII REACTION

Guo Benyu
(Shanghai University of Science and Technology, Shanghai 201800, China)
Chen Zhixiong
(Shanghai Jiaotong University, Shanghai 200030, China)
(Received Jan. 13, 1993)

Abstract Painlevé analysis is applied to analytic solutions of Noyes-Field system for Belousov-Zhabotinskii reaction. Some explicit travelling wave solutions are presented. The technique in this paper is also useful for other systems of nonlinear partial differential equations which may be integrable or nonintegrable.
Key Words Belousov-Zhabotinskii reaction; Painlevé analysis; analytic solutions.
Classification 35A05, 92A05.

1. Introduction

The Belousov-Zhabotinskii reaction is widely studied both theoretically and experimently, since it plays an important role in description of chemical reactions. A simplified reaction based on the Noyes-Field model was suggested by Murray (See [1]). By the law of mass action and an appropriate nondimensionlization, we have

$$\begin{cases} U_t = U_{xx} + U(1 - U - rV) + LrV \\ V_t = V_{xx} - bUV - MV \end{cases} \tag{1.1}$$

where $L \approx M = O(10^{-4})$, $b = O(1)$, r can be varied from 5 to 50 approximately. Since $L \ll 1$ and $M \ll 1$, we may neglect the corresponding terms and obtain the following system

$$\begin{cases} U_t = DU_{xx} + U(1 - U - rV) \\ V_t = V_{xx} - bUV \end{cases} \tag{1.2}$$

where D, r and b are positive constants. Sometimes we take $D = 1$ (See [1]).

There are lots of literature concerning travelling wave solutions of (1.2). Troy [2] showed that there exist $c^* \in (0, 2]$ and $r^* > 0$ such that if $r = r^*$ and $b \in (0, 1)$, then (1.1) and (1.2) have travelling wave solutions with the speed $c = c^*$. Murray [1, 3] did a lot of work for the models and the estimations of the wave speeds. Ye, Wang [4] and Li, Ye [5] developed a general method for analysis on the existence of such solutions, and found that if $0 < r < 1$ and $0 < b < \frac{1-r}{r}$, then (1.1) and (1.2) have travelling wave solutions, if and only if the speed $c \geq \sqrt{1-r}$. Recently, Wang, Xiong, Ye [6] proposed a new technique to obtain some explicit expressions of travelling wave solutions of (1.2).

In this paper, we shall apply Painlevé analysis to look for analytic solutions of (1.2). As we know, this approach was renewed by Ablowitz, Ramani, Segur [7] and Mcleod, Olver [8], while Weiss, Tabor, Carnevale [9] applied this technique to partial differential equations. Originally it was used to test the integrability of nonlinear systems. But it can be also used to deduce Lax-Pair, Bäcklund transformation, Hirota bilinear transformation and other properties such as symmetry (see [10]). Furthermore people found special solutions of nonlinear partial differential equations by painlevé analysis. Recently, Conte, Musette [11], Cariello, Tabor [12], Hereman [13] and the authors [14, 15] extended this approach to nonintegrable systems, and obtained many interesting solutions such as periodic solutions, travelling waves and coalescences of two waves. At the same time, the authors generalized it to systems of nonlinear partial differential equations in multi-dimensional space (see [16]).

Clearly if $U \neq 0$ and $V = 0$, then (1.2) is well-known Fisher equation (See [17]). Ablowitz Zeppetella [18] pointed out that this equation is nonintegrable, and got an explicit solution with a special wave speed. The authors [14] found several classes of solutions. Following [18], we can also verify that (1.2) is a nonintegrable system involving two nonlinear partial differential equations. We shall use Painlevé expansion to derive its analytic solutions. Two kinds of expansions are considered, which imply two classes of solutions. Many explicit expressions of travelling wave solutions are obtained. We also find travelling wave solutions with arbitrary constant speed c, provided that certain conditions are fulfilled, such as $b = \frac{1}{3D}$, $|c| > \frac{1}{5D}$. The asymptotic behaviours of these solutions are analyzed. As special cases, the results in [6] are refound. The technique used in this paper is also applicable in studying other nonintegrable systems of nonlinear partial differential equations.

2. Painleve Analysis, the First Expansion

The basic idea of Painleve analysis is the expansions about solution singularity manifold and truncation technique. Let $\phi(x,t) = 0$ be the solution singularity manifold of (1.2), $\phi_x \phi_t \neq 0$ and

$$U = \sum_{j=0}^{\infty} U_j \phi^{j-\mu}, \quad V = \sum_{j=0}^{\infty} V_j \phi^{j-\nu}$$

By substituting the above expansions into (1.2) and analysing the leading parts, we find that

$$\mu = \nu = 2, \quad \text{or} \quad \mu = 2,\ \nu = 1$$