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THE GLOBAL SOLUTION AND ASYMPTOTIC BEHAVIORS FOR ONE CLASS OF SYSTEM OF NONLINEAR EVOLUTION EQUATIONS

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Abstract The existence of global solution of initial-value problem for one class of system of nonlinear evolution equation is proved, we also study the asymptotic behavior and "blow up" of the solution.
Key Words Global solutions; evolution equations; asymptotic behaviors.
Classification 35Q.

1. Introduction and Main Results

The aim of this paper is to study the existence of global solution to the initial-value problem for the following nonlinear evolution equations

$$i\psi_t + \Delta\psi + \alpha\theta\psi + \beta|\psi|^{2p}\psi = 0, \quad x \in \mathbf{R}^n, t > 0 \tag{1}$$

$$-\Delta\theta + a^2\theta = |\psi|^2 \tag{2}$$

$$\psi(x,0) = \psi_0(x), \qquad x \in \mathbf{R}^n \tag{3}$$

where α and β are real constants, $\psi(x,t)$ is an unknown complex function, $\theta(x,t)$ is an unknown real function. $n \geq 2$. We also study the long time behavior and "blow up" of the solutions.

In the interaction of laser-plasma, the system of Zakharov equation plays an important role (See [1] [2] [3]) when the electromagnetic wave propagates in a plasma. The problem of stationary waveguide solutions arising due to thermal nonlinearities has been discussed. A thermal self focusing mechanism is quite clear. The system of equations has been proposed and studied from physics in [2].

To simplify the notation in this paper, we shall denote by $\int u(x)dx$ the integration $\int_{\mathbf{R}^n} u(x)dx$, by $\|\cdot\|_p$ the norm $\|\cdot\|_{L^p(\mathbf{R}^n)}$ by $\|\cdot\|_{m,p}$ the norm $\|\cdot\|_{W^{m,p}(\mathbf{R}^n)}$, and by C or E all the positive constants that depend only on the size of the initial data and, if necessary, on the constant T.

Our main results are as follows

Theorem 1 *Suppose that* (i) $\psi_0 \in H^m(\mathbf{R}^n)$, $n = 2,3$ *for some integer* $m \geq 2$, *and* (ii) $\beta < 0, 1 < p < 2$ *for* $n = 3$, $1 < p < \infty$ *for* $n = 2$. *Then the problem* (1)–(3) *has the solution* $\psi(x,t)$, $\theta(x,t)$ *satisfying*

$$\partial_t^r \partial_x^s \psi(x,t) \in L^\infty(0,T;L^2(\mathbf{R}^2))$$

where $2r + s \leq m$,

$$\partial_t^{r_1} \partial_x^{s_1} \theta(x,t) \in L^\infty(0,T;L^2(\mathbf{R}^n))$$

where $2r_1 + \max\{0, s_1 - 2\} \leq m$, *for any* $T > 0$.

Theorem 2 *Suppose that* $\psi_0(x) \in H^1(\mathbf{R}^n) \cap L^{2p+1}(\mathbf{R}^n) \cap L^4(\mathbf{R}^n)$, $n \geq 4$, *if one of the following conditions is satisfied:*

(1) $\alpha < 0$, $\beta \leq 0$,

(2) $\alpha < 0$, $\beta > 0$, $0 < p < \frac{2}{n}$ *or* $p = \frac{2}{n}$, $\|\psi_0\|_2 < \beta^{-\frac{n}{4}}\|\varphi\|_2$,

(3) $\alpha > 0$, $\beta < 0$, $n = 2,3$,

(4) $\alpha > 0$, $\beta > 0$, $n = 2,3$, $0 < p < \frac{2}{n}$ *or* $p = \frac{2}{n}$, $\|\psi_0\|_2 < \beta^{-\frac{n}{4}}\|\varphi\|_2$,

where φ *is the ground state solution of the following equation*

$$\Delta\varphi - \frac{2}{n}\varphi + \varphi^{\frac{4}{n}+1} = 0$$

then the problem (1)–(3) *has the solution* $\psi(x,t)$, $\theta(x,t)$ *satisfying*

$$\psi(x,t), \theta(x,t) \in L^\infty(0,T;H^1\mathbf{R}^n))$$

for any $T > 0$.

Theorem 3 *Suppose that*

(1) $\alpha < 0$, $\beta \leq 0$, $p \geq \frac{2}{n}$, $n \geq 4$,

(2) $\|x\psi_0(x)\|_2 < \infty$, $\|\psi_0(x)\|_2 < \infty$.

Then for the solution $\psi(x,t), \theta(x,t)$ *of the problem* (1)–(3) *we have*

$$\|\psi(\cdot,t)\|_q \leq Ct^{n(\frac{1}{q}-\frac{1}{2})}, \quad t > 1$$

where $2 < q \leq \frac{2n}{n-2}$,

$$\|\theta(\cdot,t)\|_q \leq Ct^{-n(1-\frac{1}{q})}, \qquad 1 \leq q \leq \frac{n}{n-2}, \quad n \geq 4$$