J.Partial Diff. Eqs. 12(1999), 345-356
©International Academic Publishers Vol.12, No.4

REDUCTION OF THE APPELL'S SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS TO THE SYSTEM OF TOTAL DIFFERENTIAL EQUATIONS

Ice B. Risteski
(Bratfordska 2/3-9, 91000 Skopje, Macedonia)
(Received Nov. 9, 1998)

Abstract In this paper it is shown that the Appell's system of partial differential equations, with two complex variables x and y, reduces to the system of total differential equations. Also, it is obtained the differential equation on the section y=const.
Key Words Reduction; Appell's system; system of total differential equations.
Classification 35A22, 35G05.

1. Introduction

Some Appell's systems of partial differential equations are considered in [1], but the conditions of the complete integrability are given in [2].

Our object of investigation in this paper will be the reduction of the Appell's system of partial differential equations to the system of total differential equations.

2. Main Results

Now we will give the following result.

Theorem *The Appell's system of partial differential equations with two complex variables x and y, given by*

$$P_1r = A_1s + B_1p + C_1q + D_1z \tag{1}$$

$$P_2t = A_2s + B_2p + C_2q + D_2z \tag{2}$$

where

$$p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y},\ r = \frac{\partial^2 z}{\partial x^2},\ s = \frac{\partial^2 z}{\partial x \partial y},\ t = \frac{\partial^2 z}{\partial y^2},\ D_i = \text{const},\ i = 1, 2$$

and polynomials with degrees

$$\deg(P_i, A_i : x, y) \le 2, \quad i = 1, 2$$

$$\deg(B_i, C_i : x, y) \le 1, \quad i = 1, 2$$

for $P_1P_2 - A_1A_2 \neq 0$, $(A_i, P_i \neq 0,\ i = 1, 2)$ *reduces to the following system of total differential equations*

$$dz_i = \sum_{j=1}^{4} h_{ij} z_j, \quad 1 \leq i \leq 4 \tag{3}$$

whose system of differential equations on the y-section is

$$\frac{dz_i}{dx} = \sum_{j=1}^{4} h'_{ij} z_j, \quad 1 \leq i \leq 4 \tag{4}$$

Proof By differentiating the Equation (1) with respect to y and Equation (2) with respect to x, we obtain the system

$$P_1\frac{\partial s}{\partial x} - A_1\frac{\partial s}{\partial y} = \Big(\frac{\partial A_1}{\partial y} + B_1\Big)s + C_1 t - \frac{\partial P_1}{\partial y} r + \frac{\partial B_1}{\partial y} p + \Big(\frac{\partial C_1}{\partial y} + D_1\Big)q + \frac{\partial D_1}{\partial y} z \tag{5}$$

$$P_2\frac{\partial s}{\partial y} - A_2\frac{\partial s}{\partial x} = \Big(\frac{\partial A_2}{\partial x} + C_2\Big)s + B_2 r - \frac{\partial P_2}{\partial x} t + \Big(\frac{\partial B_2}{\partial x} + D_2\Big)p + \frac{\partial C_2}{\partial x} q + \frac{\partial D_2}{\partial x} z \tag{6}$$

Now, we will solve the above equations (5) and (6) by $\dfrac{\partial s}{\partial x}$ and $\dfrac{\partial s}{\partial y}$, and we will express them in terms of s, p, q and z. If we put

$$P_3 = P_1P_2 - A_1A_2 \tag{7}$$

then we obtain

$$\begin{aligned}
P_3\frac{\partial s}{\partial x} =& \Big\{\Big(\frac{\partial A_1}{\partial y} + B_1\Big)P_2 + \Big(\frac{\partial A_2}{\partial x} + C_2\Big)A_1 + \Big[C_1 + A_1P_2\frac{\partial}{\partial x}\Big(\frac{1}{P_2}\Big)\Big]A_2 \\
&+ \Big[\frac{A_1}{P_1}B_2 + P_1P_2\frac{\partial}{\partial y}\Big(\frac{1}{P_1}\Big)\Big]A_1\Big\}s \\
&+ \Big\{\frac{\partial B_1}{\partial y}P_2 + \Big(\frac{\partial B_2}{\partial x} + D_2\Big)A_1 + \Big[C_1 + A_1P_2\frac{\partial}{\partial x}\Big(\frac{1}{P_2}\Big)\Big]B_2 \\
&+ \Big[\frac{A_1}{P_1}B_2 + P_1P_2\frac{\partial}{\partial y}\Big(\frac{1}{P_1}\Big)\Big]B_1\Big\}p \\
&+ \Big\{\Big(\frac{\partial C_2}{\partial y} + D_1\Big)P_2 + \frac{\partial C_2}{\partial x}A_1 + \Big[C_1 + A_1P_2\frac{\partial}{\partial x}\Big(\frac{1}{P_2}\Big)\Big]C_2 \\
&+ \Big[\frac{A_1}{P_1}B_2 + P_1P_2\frac{\partial}{\partial y}\Big(\frac{1}{P_1}\Big)\Big]C_1\Big\}q \\
&+ \Big\{\frac{\partial D_1}{\partial y}P_2 + \frac{\partial D_2}{\partial x}A_1 + \Big[C_1 + A_1P_2\frac{\partial}{\partial x}\Big(\frac{1}{P_2}\Big)\Big]D_2 \\
&+ \Big[\frac{A_1}{P_1}B_2 + P_1P_2\frac{\partial}{\partial y}\Big(\frac{1}{P_1}\Big)\Big]D_1\Big\}z \\
=& a_1 s + b_1 p + c_1 q + d_1 z
\end{aligned} \tag{8}$$