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ASYMPTOTIC BEHAVIOR OF GLOBAL SMOOTH SOLUTIONS TO THE EULER-POISSON SYSTEM IN SEMICONDUCTORS

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Abstract In this paper, we establish the global existence and the asymptotic behavior of smooth solution to the initial-boundary value problem of Euler-Poisson system which is used as the bipolar hydrodynamic model for semiconductors with the nonnegative constant doping profile.

Key Words Bipolar hydrodynamic model, semiconductors, asymptotic, smooth solution.

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1. Introduction

We are concerned with the large time behavior of smooth solutions to the one-dimensional Euler-Poisson system which is used as the bipolar hydrodynamic model for semiconductors in the case of two carriers, i.e. electron and hole. Namely

$$n_t + (nu)_x = 0, \tag{1.1}$$

$$h_t + (hv)_x = 0, \tag{1.2}$$

$$(nu)_t + (nu^2 + p(n))_x = n\phi_x - \frac{nu}{\tau_n}, \tag{1.3}$$

$$(hv)_t + (hv^2 + q(h))_x = -h\phi_x - \frac{hv}{\tau_h}, \tag{1.4}$$

$$\phi_{xx} = n - h - d(x), \tag{1.5}$$

$(t, x) \in (0, \infty) \times (0, 1)$, where (n, h) and (u, v) are densities and velocities for electrons and holes, respectively, $j = nu$ and $k = hv$ stand for the electron and hole current densities, ϕ denotes the electrostatic potential and we denote $E = \phi_x$ as the electric field, and $d(x)$ describes fixed charged background ions. The pressure functions $p(n)$ and $q(h)$ are taken as

$$p(n) = \frac{n^{\gamma_n}}{\gamma_n},\ \gamma_n > 1, \quad q(h) = \frac{h^{\gamma_h}}{\gamma_h},\ \gamma_h > 1. \tag{1.6}$$

τ_n and τ_h are the momentum relaxation times for electrons and holes, respectively. τ_n and τ_h are constants in the present paper. Furthermore, $\tau_n = \tau_h = 1$ for convenience.

Recently, the hydrodynamic model of semiconductors has attracted a lot of attention, due to its function to describe hot electron effects which are not accounted for in the classical drift-diffusion model. Rigorous results have been obtained in various papers. Most of them are concerned with the unipolar case — the case of one carrier type, i.e. electron. Also, there are a few results on the bipolar case which is of more importance and physical meaning. Fang and Ito [1] showed the existence of weak solutions to the system (1.1)-(1.5) using the viscosity argument. Natalini [6], Hsiao and Zhang [4], considered the relaxation limit problem from the bipolar hydrodynamic model to the drift-diffusion equations. Zhu and Hattori [7] showed the existence of smooth solutions to Cauchy problem of (1.1)-(1.5) and discussed the asymptotic stability of the steady state solution, when the doping profile is close to zero.

In present paper, we consider the initial boundary value problems for (1.1)-(1.5) with the following initial data

$$(n, h, j, k)(x, 0) = (n_0, h_0, j_0, k_0)(x), \quad x \in (0, 1), \tag{1.7}$$

and the insulating boundary conditions

$$j(0, t) = 0 = j(1, t), \tag{1.8}$$

$$k(0, t) = 0 = k(1, t), \tag{1.9}$$

$$E(0, t) = 0. \tag{1.10}$$

To provide some insights into the above evolutionary problem, we take the doping profile $d(x)$ as a nonnegative constant d. Our main purpose in this paper is to investigate the global existence and the asymptotic behavior of the smooth solution to (1.1)-(1.5) and (1.7)-(1.10). More precisely, we prove that when the initial data are small perturbations of a stationary solution to the system, the global smooth solution to (1.1)-(1.5) and (1.7)-(1.10) exists and tends to the stationary solution, as $t \to \infty$, exponentially. The steady state solution concerned satisfies the following system:

$$p(\bar{n})_x = \bar{n}\bar{E}, \tag{1.11}$$

$$q(\bar{h})_x = -\bar{h}\bar{E}, \tag{1.12}$$

$$\bar{E}_x = \bar{n} - \bar{h} - d, \tag{1.13}$$