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EXISTENCE OF SOLUTION FOR MODIFIED LANDAU-LIFSHITZ MODEL

Wang Liping
(Institute of Mathematics, Fudan University, Shanghai 200433, China)
(E-mail: truedrea@163.com)
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Abstract In the present paper, the existence of solutions to Cauchy problem for modified Landau–Lifshitz Model initiated by Augusto Visintin is studied.
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1. Introduction

Ferromagnetic materials can attain a large magnetization under the action of a small applied magnetic field. To explain this phenomenon, in 1907 Weiss suggested that any small portion of the body exhibits a spontaneous magnetization and is magnetically saturated even if no magnetic field is applied. In 1928 Heisenberg explained the spontaneous magnetization postulated by Weiss in terms of the exchange interaction. In 1935 Landau and Lifshitz [1] proposed a quantitative theory, now known as micromagnetics.

In the classical study of 1-dimensional motion of ferromagnetic chain, the so-called Landau-Lifshitz equation for the isotropic Heisenberg chain is a special case of the generalized systems

$$M_t = M \times M_{xx} + f(x,t,M) \tag{1.1}$$

and such an equation usually appears in the study of pure material. In the past years a lot of works contributed to the study of the soliton solution, the interaction of solitary waves and others for the Landau-Lifshitz equation in [2 − 5]. Generally speaking, the existence of global weak solutions for initial-boundary value problems and the Cauchy problem of the generalized system of ferromagnetic have been established in [6 − 8].

The system of Heisenberg spin chain

$$M_t = M \times M_{xx}, \tag{1.2}$$

also called the Landau-Lifshitz equation, is proposed to describe the evolution of spin field in continuous ferromagnets. In [9], Sulem, Sulem and Bardos studied the well-posedness for the Cauchy problem of the system (1.2). In [10], Zhou, Guo and Tan have gotten existence and uniqueness of smooth solution for the system (1.2).

The above discussion is referred to a perfect crystal and does not allow the presence of magnetic inclusions: impurities, dislocations and other defects. This also covers the case where magnetic inclusions are regularly distributed; a typical example is steel. In [11] Augusto Visintin proposed to describe the effect of defects on evolution by means of modification of the Landau-Lifshitz equation, i.e.

$$\begin{cases} M_t = M \times (M_{xx} - \frac{\eta M_t}{|M_t|}), \\ M(x,0) = M^0(x), \end{cases} \tag{1.3}$$

where η is a positive constant to account for the average distribution of defects in the material. For a nonhomogeneous material, η may depend on x , and may be also replaced by a 3×3-tensor to account for anisotropy. In this paper for simplicity $\eta =$constant is discussed. But the argument used here also works for the case $\eta = \eta(x)$.

In order to avoid singularity of (1.3) where $M_t = 0$, with $W = M_{xx} - \dfrac{\eta M_t}{\sqrt{\varepsilon^2 + \mid M_t \mid^2}}$ we first study its regularized problem

$$\begin{cases} M_t = M \times W, \\ M(x,0) = M^0(x). \end{cases} \tag{1.4}$$

Following [12], we introduce Gilbert damping to (1.4) and consider the following problem

$$\begin{cases} M_t = M \times W - \alpha M \times (M \times W), & (t,x) \in (0,T] \times \Omega, \\ M(x,0) = M^0(x), & x \in \Omega, \end{cases} \tag{1.5}$$

where T is a positive constant and $\Omega = [-1,1]$. According to the classical theory of Weiss, $|M(x,t)| = 1$. Hence (1.5) is equivalent to the following problem

$$\begin{cases} M_t = M \times W + \alpha W + \alpha |M_x|^2 M, & (t,x) \in (0,T] \times \Omega, \\ M(x,0) = M^0(x), & x \in \Omega. \end{cases} \tag{1.6}$$

Our sketch is as follows. Firstly, we establish certain a priori α-independent estimates for the solutions of the problem (1.6), which allow us to obtain a sufficient smooth solution to Cauchy problem for the problem (1.4) by passing to the limit $\alpha \to 0$. Secondly, the existence of the sufficient smooth solution for the problem (1.6) is proved by using the fixed point theorem and $\alpha-$independent estimates.

Throughout the present paper all the positive constants depending only on η, $\|M^0\|_{H^k(\Omega)}$, T, independent of α and ε, unless otherwise stated, will be denoted by C and they may be different from line to line.

Theorem *Suppose $M^0(x)$ is in $H^k(\Omega), k \geq 4$ with $|M^0(x)| = 1$ and $M^0(-1) = M^0(1)$, then for any positive constant T,*