JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **28**, No. 4, pp. 291-304

doi: 10.4208/jpde.v28.n4.1
December 2015

A Note on Singular Solutions of the Matukuma Equation in Higher Dimensional Space

WANG Biao*and ZHANG Zhengce

School of Mathematics and Statistics, Xi'an Jiaotong University, Xi'an 710049, China.

Received 7 May 2015; Accepted 29 August 2015

Abstract. We revise one case of the M-solutions of B. Wang, et al., J. Diff. Eqs. 253 (2012), pp. 3232-3265 and obtain more precise form of the singular term and the regular term.

AMS Subject Classifications: 35J15, 35J61, 35B40

Chinese Library Classifications: O175.25, O175.29

Key Words: Matukuma equation; singular solutions; asymptotic expansion.

---

1 Introduction

In this paper, we review one case of the M-solutions of positive radial solutions $\phi=\phi(r)$ of the differential equation

$$\frac{1}{r^{N-1}}\left(r^{N-1}\phi'\right)'=-\frac{r^{\lambda-2}}{(1+r^2)^{\lambda/2}}\phi^p, \quad p>1,\ \lambda>0,\ N>3. \tag{1.1}$$

In [1], in order to apply a theorem on asymptotically autonomous systems by Thieme [2], Emden-Fowler equation was extended into the following generalized one

$$\frac{1}{r^{N-1}}\left(r^{N-1}\phi'\right)'=-r^{q-N}\phi^p, \quad p>1,\ q>N-2,\ N>3. \tag{1.2}$$

By using the substitution

$$u(t)=r^{q-N+1}\frac{\phi^p(r)}{-\phi'(r)}, \qquad v(t)=r\frac{-\phi'(r)}{\phi(r)}, \qquad r=e^t,$$

*Corresponding author. *Email addresses:* `wang.biao@stu.xjtu.edu.cn` (B. Wang), `zhangzc@mail.xjtu.edu.cn` (Z.C. Zhang)

one could transfer the solutions of (1.2) into the solutions $\varphi=(u,v)$ of the Lotka-Volterra system

$$\begin{cases} \dot{u}=u(q-u-pv), \\ \dot{v}=v(-N+2+u+v). \end{cases} \tag{1.3}$$

Applying another substitution (see [1] for more details)

$$u(t)=\frac{r^{\lambda-1}}{(1+r^2)^{\lambda/2}}\frac{\phi^p(r)}{-\phi'(r)}, \qquad v(t)=r\frac{-\phi'(r)}{\phi(r)}, \qquad r=e^t,$$

to solutions of (1.1), we get

$$\begin{cases} \dot{u}=u(q(t)-u-pv), \\ \dot{v}=v(-N+2+u+v). \end{cases} \tag{1.4}$$

Here the coefficient

$$q(t)=N-2+\frac{\lambda}{1+r^2}, \quad N>3,$$

is time-dependent, but the limits

$$\lim_{t\to-\infty}q(t)=N-2+\lambda, \qquad \lim_{t\to+\infty}q(t)=N-2$$

exist, and we define $q:=N-2+\lambda$. Then (1.4) is asymptotically autonomous with respect to (1.3) for $t\to-\infty$ and to $(EFS_{p,N-2,N})$ for $t\to+\infty$, and this fact makes this system an asymptotically autonomous one.

Batt and Li [3] developed a comprehensive theory on all positive solutions ϕ of the Matukuma Eq. (1.1) in $\mathbb{R}^3$. They classified the solutions of the Matukuma equation into three different types which known exists for the Emden-Fowler Eq. (1.2), namely, the E-solutions (regular at $r=0$), the F-solutions (whose existence begins away from $r=0$) and the M-solutions (singular at $r=0$) and obtained all kinds of properties of those three solutions. Wang et al. [1] extended the work of Batt and Li [3] into the higher dimensional space and investigated the properties of the M-solutions. Their result shows that the M-solutions in higher dimensional space is dramatically different to that of $N=3$. In this paper, we continue to study the M-solutions of (1.1) in higher dimensional space. The basic tool is an asymptotically expansionary method which begins with rough estimates and improves the accuracy step by step to the desired extent [4,5]. It is known [1] that when $q>(N-2)p$, the M-solutions possess a splitting form: $\phi(r)=S+\Theta$, where S is a singular term of form $S=\frac{c}{r^{N-2}}P(r)$ with an elementary explicitly given function P of r with $P(r)=1+o(1)(r\to 0)$, whereas Θ is a regular solution of the initial value problem

$$\begin{cases} \frac{1}{r^{N-1}}\Big(r^{N-1}\Theta'\Big)'=-\frac{r^{\lambda-2}}{(1+r^2)^{\lambda/2}}\Big(\Theta+S\Big)^p-\frac{1}{r^{N-1}}\Big(r^{N-1}S'\Big)', \qquad 0<r<R, \\ \Theta(0):=\lim\limits_{r\to 0}\Theta(r)=\beta\in\mathbb{R}. \end{cases} \tag{1.5}$$