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Pullback Dynamics of 2D Non-autonomous Navier-Stokes Equations with Klein-Voight Damping and Multi-delays

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Abstract. This paper is concerned with the pullback dynamics of 2D non-autonomous Navier-Stokes-Voigt equations with continuous and distributed delays on bounded domain. Under some regular assumptions on initial and delay data, the existence of evolutionary process and the family of pullback attractors for this fluid flow model with Klein-Voight damping are derived. The regular assumption of external force is less than [1].

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1 Introduction

The incompressible Navier-Stokes equations play an important role in hydrodynamical systems which describes the essential law. Our purpose of this paper is to investigate the pullback dynamics for the 2D non-autonomous Navier-Stokes equations with

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Klein-Voight damping (also as Navier-Stokes-Voight equations) and double delays on a bounded domain Ω with smooth boundary as following:

$$\begin{cases} u_t-\nu\Delta u-\alpha^2\Delta u_t+(u\cdot\nabla)u+\nabla p=f\Big(t-\rho(t),u(t-\rho(t))\Big) \\ \qquad\qquad\qquad\qquad\qquad\qquad +\int_{-h}^{0}G(s,u(t+s))\mathrm{d}s, \qquad \text{in } \Omega\times(\tau,+\infty), \\ \mathrm{div}u=0, \qquad \text{in } \Omega\times(\tau,+\infty), \\ u=0, \qquad \text{on } \partial\Omega\times(\tau,+\infty), \\ u(\tau,x)=u^{\tau}(x), \qquad \text{in } \Omega, \\ u(t,x)=\phi(t-\tau,x), \qquad \text{in } \Omega\times(\tau-h,\tau), \end{cases} \tag{1.1}$$

here $\alpha>0$ is a length scale parameter, the kinematic viscosity $\nu>0$, p is the pressure, $f(t-\rho(t),u(t-\rho(t)))$ denotes the continuous delay external force, $g(t,u_t)=\int_{-h}^{0}G(s,u(t+s))\mathrm{d}s$ is the distributed delay external forcing term, $u_t(s)=u(t+s)$, $s\in(-h,0)$, $h>0$ is a constant, and ϕ is the initial data of delay in $[\tau-h,\tau]$.

The Navier-Stokes-Voigt system is an approximation Navier-Stokes system and models the dynamics of a Kelvin-Voigt viscoelastic incompressible fluid which was introduced by Oskolkov [2]. Some results of Navier-Stokes-Voigt equations can be found in Kalantarov [3], Kalantarov, Levant and Titi [4], Gao and Sun [5], Sun and Gao [6], Kalantarov and Titi [7,8], Qin, Yang and Liu [9], Çelebi, Kalantarov and Polat [10] and so on.

The delay phenomenon describes some interesting law in economics, autonomic science and engineer, biology and so on. In 2002, Krasovskii [11] first noticed the system with delay, constructed the Navier-Stokes equations with delay, and obtained the well-posedness of the system. Hale [12] established some conclusions on the differential equations with delay and established the existence and uniqueness of weak solutions to the Navier-Stokes equations with delay, which laid the foundation of dynamic system of the Navier-Stokes equations with delay. On an open and bounded domain with regular boundary, Caraballo and Real [13–15] investigated the Navier-Stokes equations with delay and derived some results such as the well-posedness, the asymptotic behavior of solutions, the existence of pullback attractors and so on. Garcín-Luengo, Marín-Rubio and Planas [16] studied the 2D Navier-Stokes equations with double delays, and derived the existence of pullback attractors. In 2015, Keqin Su *et al* [1] considered the Navier-Stokes-Voigt equations with double delay on a non-smooth domain, and obtained the existence of bounded family of pullback attractors. Other results about the fluid flow with delays, one can refer to [17–24].

In this paper, under less regularity on the delay external force $f(t-\rho(t),u(t-\rho(t)))$, the pullback dynamics of problem (1.1) on bounded domain with smooth boundary will be investigated which is more general than [1]. This paper is organized as follows. In Section 2, some preliminaries are given which will be used in sequel. In Sections 3 and 4, the well-posedness of solutions, the existence of processes, pullback absorbing balls and the existence of pullback attractors are proved.