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Finite-Volume TENO Scheme with a New Cell-Interface Flux Evaluation Strategy for Unstructured Meshes

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Abstract. The development of high-order shock-capturing schemes is critical for compressible fluid simulations, in particular for cases where both shock waves and small-scale turbulence structures present. As one of the state-of-the-art high-order numerical schemes, the family of high-order targeted ENO (TENO) schemes proposed by Fu et al. [Journal of Computational Physics 305 (2016): 333-359] has been demonstrated to perform well for compressible gas dynamics on structured meshes and recently extended to unstructured meshes by Ji et al. [Journal of Scientific Computing 92(2022): 1-39]. In this paper, with the observation that the TENO scheme not only provides the high-order reconstructed data at the cell interface but also features the potential to separate the local flow scales in the wavenumber space, we propose a low-dissipation finite-volume TENO scheme with a new cell-interface flux evaluation strategy for unstructured meshes. The novelty originates from the fact that the local flow scales are classified, following a strong scale separation in the reconstruction process, as "very smooth" or not. When the corresponding large central-biased stencil for the targeted cell interface is judged to be "very smooth", a low-dissipation Riemann solver, even the non-dissipative central flux scheme, is employed for the cell-interface flux computing. Otherwise, a dissipative approximate Riemann solver is employed to avoid spurious oscillations and achieve stable shock-capturing. Such a strategy provides separate control over the numerical dissipation of the high-order reconstruction process and the cell-interface flux calculation within a unified framework and leads to a resultant finite-volume method with extremely low-dissipation properties and good numerical robustness. Without parameter tuning case by case, a set of canonical benchmark simulations has been conducted to assess the performance of the proposed scheme.

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1 Introduction

For computational fluid dynamics (CFD) governed by a system of hyperbolic conservation laws, the development of high-order stable numerical methods on unstructured mesh has been a long-term challenging research topic and plays an important role in modern engineering industries. The catalyst for this development is that mixed-element unstructured meshes can provide significant convenience for computational workflows that involve complex geometries. The primary design objectives for modern numerical methods are that the schemes should achieve the high-order spatial accuracy in smooth flow regions and have sufficient dissipation to preserve the non-oscillatory property in the vicinity of discontinuities [1, 2]. However, when considering complex fluid simulations with broadband flow length scales, the low-dissipation property is also critical for resolving the small-scale flow structures with high spectral resolution, especially for the under-resolved large-eddy simulations (LES) deployed on coarse meshes. Finding the optimal balance between these competing requirements has been the primary driving force and challenge behind developing the modern numerical methods [3–5]. Well-established shock-capturing concepts have been proposed over the last decades, e.g., the total variation diminishing (TVD) schemes [6–9], the essentially non-oscillatory (ENO) schemes [10] and the weighted ENO (WENO) [11] schemes.

As the most popular arbitrarily high-order numerical methods, the WENO-family schemes, established based on the ENO shock-capturing concept [10], are first proposed by Liu et al. [11] in the finite-volume framework and then extended to the finite-difference framework by Jiang and Shu [1]. Afterwards, lots of variants are proposed and developed to further improve the performance of classical WENO-JS schemes, e.g., the WENO-M [12] scheme, which remedies the accuracy order degradation of the WENO-JS scheme at the critical point by remapping the nonlinear weights, the WENO-Z [13] scheme, which introduces the novel weighting strategy based on a global smoothness indicator to satisfy the sufficient criteria of high accuracy order, the WENO scheme with modified weighting strategy [14], the hybrid WENO scheme [15], etc. While significant success has been achieved with these WENO variant schemes on structured meshes due to the high-order convergence in smooth regions and the sharp shock-capturing property in the vicinity of discontinuities [1,16], it is, however, challenging to deploy the WENO-family schemes to unstructured triangular or tetrahedral meshes.

The main issue is that the optimal linear weights for constituting the high-order reconstruction by the combination of small stencils may vary considerably for different mesh topologies and different Gaussian integration points, and even become negative,