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REVIEW ARTICLE

A Comparative Study of Hydrodynamic Lattice Boltzmann Equation in Phase-Field-Based Multiphase Flow Models

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Abstract. In recent years, phase-field-based models for multiphase flows have gained significant popularity, particularly within the lattice Boltzmann (LB) community. These models typically use two lattice Boltzmann equations (LBEs), one for interface tracking and the other for solving hydrodynamic properties. However, for the purposes of this paper, we focus only on the LB model for hydrodynamics. Our goal is to undertake a comparative investigation into the differences between three classical hydrodynamic LB models proposed by Lee et al. [1], Liang et al. [2] and Fakhari et al. [3]. The interface-tracking equation used in this study is based on the conservative phase-field model. We provide a detailed derivation of the governing equations in each model using the Chapman-Enskog analysis. Additionally, three discretization methods for the interaction forces are introduced, and a modified method for the gradient term is proposed based on the nonequilibrium distribution method. The accuracy of three LB models in combination with four discretization methods is examined in this study. Based on the results, it appears that different combinations of models and methods are appropriate for different types of problems. However, some suggestions for the selection of hydrodynamic models and discrete methods for the gradient term are provided in this paper.

AMS subject classifications: 76D05, 76D45, 76T10

Key words: Lattice Boltzmann method, multiphase, phase-field method, hydrodynamic lattice Boltzmann equation.

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1 Introduction

Modeling multiphase flows is crucial for many engineering and scientific applications, but it remains a challenging task due to the complex dynamics of interfacial interactions between different phases [4, 5]. In the past three decades, the lattice Boltzmann (LB) method, based on the mesoscopic kinetic theory, has been developed to be a powerful tool for simulating multiphase flows [4, 6]. Up to now, multiphase LB models can be divided into four categories, namely color-gradient models [7], pseudopotential models [8, 9], free-energy models [10], and phase-field models [11]. Among the models considered, the phase-field lattice Boltzmann (LB) method has garnered considerable attention in recent times. This is primarily attributed to its remarkable numerical stability and accuracy when addressing complex multiphase flow problems, especially those involving large density and viscosity ratios [2, 3, 12, 13]. Consequently, it has found widespread utilization in various applications, including but not limited to droplet dynamics [14], bubble dynamics [15], and multiphase flows involving multiphysics fields [16–18].

In the phase-field-based models, two lattice Boltzmann equations are utilized, one for hydrodynamics and the other for solving the convection-diffusion equation for the phase field [4]. The first consistent multiphase LBM based on the Cahn-Hilliard (C-H) equation, which is the phase field equation used to track the interface, was proposed by He, Chen, and Zhang [11]. However, this model was limited to density ratios up to approximately 15, as reported in their study of Rayleigh-Taylor instability [19]. Inamuro et al. [20] developed a phase-field-based model that can simulate three-dimensional multiphase flows with large density ratios, but the model required solving a time-consuming pressure-Poisson equation to enforce the incompressible condition. Lee and Lin [21] proposed a three-stage stable discretization multiphase LB scheme for high-density ratio flows, utilizing a second-order biased/mixed difference for the forcing terms in the pre-streaming collision step and a standard central difference in the post-streaming collision step for treating the force terms. However, this model was criticized for its complexity and time-consuming nature, as well as its incapability to recover the correct interface-capturing equation and violation of mass conservation. Lee et al. [1] demonstrated that the three-step algorithm could be simplified to the conventional two-step collision-streaming approach without compromising accuracy or stability. However, the mixed difference scheme was retained in their model, resulting in the total mass being not strictly conserved [22]. Zheng et al. [23] presented a modified LB model for multiphase flows, claiming that it could handle high-density ratio problems. However, it was later shown that the model was only suitable for density-matched binary fluids [24]. Liang et al. [25] proposed a phase-field-based multiple-relaxation-time model for incompressible multiphase flows, incorporating a time-dependent source term into the interfacial evolution equation to accurately recover the Cahn-Hilliard equation. All the aforementioned models utilize the Cahn-Hilliard equation as the governing equation of the phase field, which contains a fourth-order spatial derivative term.

Recently, the conservative Allen-Cahn (A-C) equation has emerged as a simpler al-