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Dynamics Analysis for a Hierarchical System with Nonlinear Cut-Off Interaction and Free Will

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Abstract. Noting that the particles communicate with each other within a finite distance, in the paper, we investigate the hierarchical model with free will and cut-off function (linear or nonlinear). As our observation, the cut-off size r is a sensitive coefficient to achieve the flocking behavior. Besides we will use energy function to achieve some sufficient conditions to achieve a flock. The free-will represents the tendency of the particle itself to move. It will either facilitate or prevent cluster generation. Some numerical simulations will validate our conclusions.

Key Words: Motsch-Tadmor model, hierarchical structure, free will, flocking behavior.

AMS Subject Classifications: 34M03, 70E55

1 Introduction

In recent years, cluster phenomenon has attracted more and more researchers' attention. For example, migrating birds, schools of fish regroup after encountering predators, honeybee colonies that gather in nature, and so on. Scientists use scientific means to study its internal dynamics law, and have achieved fruitful results. The terminology flocking represents the phenomenon that self-propelled agents achieve a consentaneous velocity asymptotically.

Since 1995 when Vicsek [1] proposed an interactive dynamics model of N particles, this work has been favored by many researchers. Among them, Cucker and Smale [2] extended models have attracted a lot of attention from scholars, which is given by

$$\begin{cases} \dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \\ \dfrac{d\mathbf{v}_i}{dt} = \dfrac{\lambda}{N}\displaystyle\sum_{j=1}^{N} \varphi(|\mathbf{x}_j - \mathbf{x}_i|)(\mathbf{v}_j - \mathbf{v}_i), \quad i = 1, 2, \cdots, N. \end{cases} \tag{1.1}$$

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Here λ is a positive constant denoting the coupling strength and $\varphi(\cdot)$ is the interaction function. This model resembles a Newton-type N-body system for an interacting particle system. A classical conclusion about $1/2$ is presented. It should be noted that the interchange function $\varphi(\mathbf{x})$ exists in the whole process in the C-S model, i.e.,

$$\varphi(\mathbf{x}) = \frac{1}{(1+||\mathbf{x}_i(t)-\mathbf{x}_j(t)||^2)^\beta}.$$

However, in the above model, the interaction function of particles still exists with the increase of distance. If a small group moves away from a much larger group, it is found that the dynamics of the small group is almost masked by $\frac{1}{N}$. So to remedy this deficiency, Motsch and Tadmor [3] proposed the following model:

$$\begin{cases} \dfrac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \\ \dfrac{d\mathbf{v}_i}{dt} = \dfrac{\lambda}{\sum_{j=1}^N \psi(|\mathbf{x}_j-\mathbf{x}_i|)} \sum_{j=1}^N \psi(|\mathbf{x}_j-\mathbf{x}_i|)(\mathbf{v}_j-\mathbf{v}_i), \quad i=1,2,\cdots,N. \end{cases} \tag{1.2}$$

In the Motsch-Tadmor model (1.2), ψ stands for communication function. Hence, it is of certain significance to study this paper based on M-T model due to the relative distance. Furthermore, the researchers considered multi-particle models with realistic delay factors and new analysis methods [5–13]).

In fact, the function ψ in the above model is global in space. But the reality is the case that the impact function should be falling sharply at some distance. The Vicsek model captures this property, i.e., local interaction ($\chi_r(s)$). In this paper, $\chi_r(s)$ is defined as:

$$\chi_r(s) = \begin{cases} \varphi(s), & |s|<r, \\ 0, & |s|\geq r. \end{cases} \tag{1.3}$$

In $\chi_r(s)$ (1.3), $\varphi(s)=1$ or $(1+s^2)^{-\beta}$, s is the displacement difference between the particles. r stands for maximum communication distance, and communication is interrupted when the distance between particles exceeds r. When $r=\infty$,

$$\varphi(\mathbf{x}) = (1+||\mathbf{x}_i(t)-\mathbf{x}_j(t)||^2)^{-\beta},$$

it is the classical CS model (1.1). When $r=0$, it is a trivial solution and the clustering depends strictly on the initial value.

In 2003, Scaruffi revealed the existence of free will in the insect world, listing and particles in his monograph. Then biologists disclosed that from single-celled tissue to human beings has free will. In the existing models, little attention has been paid to the intrinsic dynamics of particles, which means the whole flock eventually moves along a straight-line direction. In motion, however, a shoal of fish or a flock of birds takes on a more complex trajectory. This is because each particle is affected not only by its neighbours, but also due to an intrinsic dynamical process. So it makes sense to explore the