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The Mean Time to Absorption on Horizontal Partitioned Sierpinski Gasket Networks

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Abstract. The random walk is one of the most basic dynamic properties of complex networks, which has gradually become a research hotspot in recent years due to its many applications in actual networks. An important characteristic of the random walk is the mean time to absorption, which plays an extremely important role in the study of topology, dynamics and practical application of complex networks. Analyzing the mean time to absorption on the regular iterative self-similar network models is an important way to explore the influence of self-similarity on the properties of random walks on the network. The existing literatures have proved that even local self-similar structures can greatly affect the properties of random walks on the global network, but they have failed to prove whether these effects are related to the scale of these self-similar structures. In this article, we construct and study a class of Horizontal Partitioned Sierpinski Gasket network model based on the classic Sierpinski gasket network, which is composed of local self-similar structures, and the scale of these structures will be controlled by the partition coefficient k. Then, the analytical expressions and approximate expressions of the mean time to absorption on the network model are obtained, which prove that the size of the self-similar structure in the network will directly restrict the influence of the self-similar structure on the properties of random walks on the network. Finally, we also analyzed the mean time to absorption of different absorption nodes on the network to find the location of the node with the highest absorption efficiency.

Key Words: Mean time to absorption, self-similar network, Sierpinski Gasket.

AMS Subject Classifications: 05C81, 05C82, 05C72, 05C76

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1 Introduction

Due to its applications in the fields of social sciences, engineering, telecommunication networks and biological networks, complex network science has gradually become a research hotspot in recent years [1–4]. The random walk on the network is a research direction in the field of complex networks that has attracted much attention, because it can intuitively describe the dynamics of complex networks [5]. In the field of complex networks, the research of random walk is generally used to detect the community structure in the network [6], to segment the network [7] and to study the corresponding properties of the resistance network [5], and so on. In addition, the random walk on complex networks has its application value in many practical fields. Current research has shown that the related properties of random walks can be used in the field of communication and information to study a series of issues such as information transmission [8], data collection [9, 10], communication quantification and prediction [11, 12], information latency [8], communication and search costs [13, 14], and computer vision [15]; in the field of biology, random walk is used to model and study the spread of infectious diseases and the metabolic flux of organisms [15, 16]. The most basic problem in the properties of random walk is the first passage time (FPT), which is defined as the number of steps required by the initial node in the network to reach the target node for the first time after random walks [5]. Since there may be many paths between these two nodes and randomness of walking, the first passage time is uncertain. Naturally, the mean first passage time(MFPT) between two nodes has received more attention, which has important application value in studying the transmission cost of wireless networks [11, 17]. But the mean first passage time only describes the local information of the network. In order to reveal the global random walk properties of the network, the mean time to absorption (MTA) is further developed on the basis of it, which is defined as the average value of the mean first passage time of all nodes in the network to the absorptive node [18, 19]. MTA directly reflects the efficiency of other nodes in the network to reach absorptive node through random walks and therefore plays an important role in the selection of data collection nodes and best absorptive sites for exciton transport in polymer and electron transfer on a fractal photosynthetic antenna [20–27].

The analytical expression of MTA on a general random network is difficult to be obtained, but on a regular iterative network with a specific structure, it is possible to find the iterative expression of MTA according to the iterative law of the network structure, which is of great significance for further understanding and studying the influence of network structure on random walk [28–30]. Sierpinski gasket network is a classic self-similar network model, so many scholars analyzed the influence of self-similarity on the topology and dynamics of the network by studying this network and its extended network model [19, 31–40]. For example, the MTA and the spectrum problem with absorptive nodes on the second-order Sierpinski gasket have been studied by Kozak et al. [41, 42]; then, Zhang et al. made certain improvements to the classic Sierpinski gasket network, and obtained network models with more specific properties, including: de-