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Variational Low-Light Image Enhancement Based on Fractional-Order Differential

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Abstract. Images captured under insufficient light conditions often suffer from noticeable degradation of visibility, brightness and contrast. Existing methods pose limitations on enhancing low-visibility images, especially for diverse low-light conditions. In this paper, we first propose a new variational model for estimating the illumination map based on fractional-order differential. Once the illumination map is obtained, we directly inject the well-constructed illumination map into a general image restoration model, whose regularization terms can be viewed as an adaptive mapping. Since the regularization term in the restoration part can be arbitrary, one can model the regularization term by using different off-the-shelf denoisers and do not need to explicitly design various priors on the reflectance component. Because of flexibility of the model, the desired enhanced results can be solved efficiently by techniques like the plug-and-play inspired algorithm. Numerical experiments based on three public datasets demonstrate that our proposed method outperforms other competing methods, including deep learning approaches, under three commonly used metrics in terms of visual quality and image quality assessment.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07
Key words: Low-light image, image enhancement, fractional-order, variational methods.

1 Introduction

Image enhancement refers to the process of highlighting certain information of an observed low-visibility image, as well as weakening or removing any unnecessary information according to specific needs [1]. With the prevalence of webcams and camera phones,

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the problem of image enhancement has become extremely challenging as images captured by digital devices may influenced by various conditions such as insufficient light, bad weather, unknown noise and so on. The enhanced image quality directly affects high-level image analysis and understanding, which is widely used in many scientific, engineering and medical applications. Thus, developing advanced image enhancement techniques are of great significance and urgency.

Retinex theory is first introduced to model the perception of human vision and is used to remove illumination effects [2]. According to the basic assumption of the Retinex theory [3, 4], an observed image can be decomposed into the illumination and reflectance components. Due to the ill-posedness of the decomposition problem, numerous approaches [5–7] have been proposed to mitigate the degradation caused by low-light conditions. These methods of low-light enhancement are mainly divided into three categories. Roughly speaking, the first one is two-step methods, which estimate the illumination map and then make use of image restoration methods to recover the desired light-enhanced scene. For example, LIME [8] first estimate the illumination map by using a structure prior and the enhancement can be achieved by using gamma transformation. The second method is the joint filtering method [9]. This kind of method aims to transfer the important structural details of the guidance to the target image in the filtering process. The third method is to estimate illumination map and reflectance component simultaneously by solving a joint minimization problem with image priors on both them. The image priors include sparsity regularization [10], fractional-order regularization [11], high-order total variation regularization [12, 13] and low rank prior [14].

Variational interpretations of Retinex are the set of enhancement algorithms [15, 16]. The first variational framework was proposed by Kimmel et al. [17] to estimate the illumination, and its objective function is established based on the smooth illumination assumption. In [8], the illumination map was first constructed by finding the maximum intensity of each pixel in all channels (i.e. bright channel). Then, the initial illumination map is refined by adding the L_1 norm on the first order derivative of the illumination. Afterwards, bright channel prior has been used in other works [18, 19] to eliminate the black halo and suppress color distortion. However, it always encounters the problems of dispersion of light in light dominant areas, over enhancement in bright regions and unwanted artifacts. Li et al. [20] used L_1 norm to constrain the piece-wise smoothness of the illumination, and adopted a fidelity term between the gradient of the reflectance and an adjusted version of the gradient of the input image, so that the structural information of the reflectance can be strengthened. Gu et al. [21] performed the fractional-order gradient total variation regularization on both the reflectance and illumination components to control the regularization extent more flexibly. Park et al. [22] proposed a L_2 norm minimization based a variational Retinex model by using a spatially adaptive weight map, which is generated by combining the local variance map and bright channel prior [23]. Gu et al. [24] performed L_2 norm on the gradient of the illumination and L_1 norm on the reflectance in the image domain along with a fidelity term to estimate the illumination and the reflectance simultaneously. Ren et al. [25] proposed a reasonable camera