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An Arbitrary Lagrangian-Eulerian Discontinuous Galerkin Scheme for Compressible Multi-Material Flows on Adaptive Quadrilateral Meshes

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Abstract. In this paper, a direct arbitrary Lagrangian-Eulerian (ALE) discontinuous Galerkin (DG) scheme is proposed for simulating compressible multi-material flows on the adaptive quadrilateral meshes. Our scheme couples a conservative equation related to the volume-fraction model with the Euler equations for describing the dynamics of the fluid mixture. The coupled system is discretized in the reference element and we use a kind of Taylor expansion basis functions to construct the interpolation polynomials of the variables. We show the property that the material derivatives of the basis functions in the DG discretization are equal to zero, with which the scheme is simplified. In addition, the mesh velocity in the ALE framework is obtained by using the adaptive mesh method from [H.Z. Tang and T. Tang, Adaptive mesh methods for one-and two-dimensional hyperbolic conservation laws, SIAM J. NUMER. ANAL]. This adaptive mesh method can automatically concentrate the mesh nodes near the regions with large gradient values and greatly reduces the numerical dissipation near the material interfaces in the simulations. With the help of this adaptive mesh method, the resolution of the solution near the target regions can be greatly improved and the computational efficiency of the simulation is increased. Our scheme can be applied in the simulations for the gas and water media efficiently, and it is more concise compared to some other methods such as the indirect ALE methods. Several examples including the gas-water flow problem are presented to demonstrate the efficiency of our scheme,

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and the results show that our scheme can capture the wave structures sharply with high robustness.

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Key words: Compressible multi-material flows, Arbitrary Lagrangian-Eulerian, Adaptive quadrilateral meshes, Discontinuous Galerkin scheme, Volume-fraction model.

1 Introduction

The hydrodynamics of multi-material flows such as gas and water is of great interest in Computational Fluid Dynamics (CFD) and exists in many problems such as the underwater explosion and the Inertial Confinement Fusion (ICF), etc. It has a wide application background in the fields of national economy and energy, etc. Its simulation has always been one of the difficult and frontier problems in the field of the fluid simulation. The simulation of compressible multi-material flows has the great theoretic significance and application value for understanding the physical phenomena in nuclear physics, biological engineering, and many other research fields. So, the research for the numerical simulations of compressible multi-material flows has obtained more and more attention from the scholars in recent years.

The multi-material fluid flows have some challenging problems in both theory and numerical simulations. The Eulerian method and the Lagrangian method are two kinds of classical methods used for dealing with multi-material flows. The Eulerian method [1–4] has strong robustness for solving the cases with large deformations, and the high-resolution schemes like essentially non-oscillatory (ENO) schemes, etc. [5, 6] based on the Eulerian method perform well when they are applied into the simulations of the single-material flow. However, when these algorithms are applied into the simulations of the multi-material cases, due to the numerical inaccuracies caused by the transport calculation, it is quite difficult for them to capture the precise physical interfaces. The Lagrangian method [7–14] has been studied by many scholars such as Després [7], Maire [8, 12], Rieben [10, 11] and Shashkov [13, 14], et al. This method can catch the material interfaces clearly in the simulations of multi-material flows. However, the large mesh distortions may lead to the interruptions of the computational codes or some errors in the simulations with large deformations, and these schemes need to introduce the mesh rezoning phase and the variables remapping phase for avoiding the large mesh distortions. For combining the advantages of this two kinds of methods above, Hirt et al. [15] has presented an arbitrary Lagrangian-Eulerian (ALE) method whose mesh nodes can move with the arbitrary velocity. The ALE method can flexibility simulate the flows with large deformations and moving regions. There are two kinds of ALE methods. The first one called indirect ALE method [16–18] consists of three steps: a Lagrangian step, a mesh rezoning step, and a variables remapping step. The second one called direct ALE method [19–21] consists of two steps: the step for obtaining the mesh velocity, the step