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An Adaptive Method Based on Local Dynamic Mode Decomposition for Parametric Dynamical Systems

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Abstract. Parametric dynamical systems are widely used to model physical systems, but their numerical simulation can be computationally demanding due to nonlinearity, long-time simulation, and multi-query requirements. Model reduction methods aim to reduce computation complexity and improve simulation efficiency. However, traditional model reduction methods are inefficient for parametric dynamical systems with nonlinear structures. To address this challenge, we propose an adaptive method based on local dynamic mode decomposition (DMD) to construct an efficient and reliable surrogate model. We propose an improved greedy algorithm to generate the atoms set Θ based on a sequence of relatively small training sets, which could reduce the effect of large training set. At each enrichment step, we construct a local sub-surrogate model using the Taylor expansion and DMD, resulting in the ability to predict the state at any time without solving the original dynamical system. Moreover, our method provides the best approximation almost everywhere over the parameter domain with certain smoothness assumptions, thanks to the gradient information. At last, three concrete examples are presented to illustrate the effectiveness of the proposed method.

AMS subject classifications: 35R60, 60H35, 65M99, 68W99

Key words: Parametric dynamical systems, dynamic mode decomposition, greedy algorithm, Taylor expansion, surrogate model.

1 Introduction

Parametrized partial differential equations arise in various engineering and applied science problems, including heat and mass transfer, acoustics, solid and fluid mechanics,

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electromagnetics, and finance. Due to limited knowledge about physical properties, such as material properties and geometric features, the model inputs often contain parameters or uncertainties. Estimating the unknown parameter values and quantifying their effects usually require a large number of realizations, ranging from thousands to millions. Consequently, traditional full-order techniques, such as finite element or finite volume methods, are often too computationally expensive, especially when dealing with nonlinear, multiphysics, and time-dependent phenomena. To overcome these challenges, many model order reduction (MOR) methods have been developed to construct efficient surrogate models, including generalized Polynomial chaos expansion [1, 2], tensor decomposition-based methods (e.g., the Proper Generalized Decomposition [3, 4] and the Variable-separation method [5–7]), and projection-based methods (e.g., the Reduced Basis method [8–10] and the Proper orthogonal decomposition (POD) Galerkin method [11, 12]). These MOR methods aim to reduce computational costs while accurately capturing the most important features of the original system.

MOR methods aim to construct an approximate model in a low-dimensional subspace of the solution space [13–16]. The success of these methods relies on the assumption that the solution manifold can be embedded in a low-dimensional space. However, the important class of problems given by parametric dynamical systems usually induce rough solution manifold with slowly decaying Kolmogorov n-widths. This implies that traditional MOR methods are generally not effective. In recent years, there has been a growing interest in the development of MOR techniques for parametric dynamical systems to overcome the limitations of linear global approximations. A large class of methods consider the dynamical low rank (DLR) approximation (see [17–21]) which allows both the deterministic and stochastic basis functions to evolve in time. Other strategies based on deep learning (DL) algorithms were proposed in [22–24] to construct the efficient surrogate model for time-dependent parametrized PDEs. In this contribution, we try to combine dynamic mode decomposition with the local Taylor approximation to construct an efficient and reliable approximation of input-output relationship (i.e. surrogate model) for parametric dynamical systems.

Data driven methods have received widespread attention. Koopman operator [25] can be an effective data driven tool. It can transform a nonlinear system in state space into a linear system in observation function space. Koopman operator is an infinite dimensional linear operator acting on the observation function space. The spectral decomposition of the Koopman can capture linear systems in the observation function space. For numerical computation, it is necessary to approximate the Koopman operator in a finite dimensional subspace. Dynamic mode decomposition is used to approximate Koopman eigenvalues and eigenvectors in the subspace are spanned by a set of observation functions. DMD [26] describes the dynamical system in an equation-free manner and can be used for prediction and control. DMD is a spatio-temporal matrix decomposition method that connects spatial dimensionality-reduction technology and Fourier transforms in time. In the standard DMD method [26], identity functions are used as a finite dimensional set of observation functions for approximate the Koopman operator.