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On a Partially Non-Stationary Vector AR Model with Vector GARCH Noises: Estimation and Testing

Chor-yiu Sin[1], Zichuan Mi[2] and Shiqing Ling[3,*]

[1] *Department of Economics, National Tsing Hua University, Hsinchu 30013, Taiwan, P.R. China.*
[2] *School of Statistics, Shanxi University of Finance and Economics, Taiyuan 030006, P.R. China.*
[3] *Department of Mathematics, The Hong Kong University of Science and Technology, Kowloon, Hong Kong, SAR, P.R. China.*

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Abstract. This paper studies a partially nonstationary vector autoregressive (VAR) model with vector GARCH noises. We study the full rank and the reduced rank quasi-maximum likelihood estimators (QMLE) of parameters in the model. It is shown that both QMLE of long-run parameters asymptotically converge to a functional of two correlated vector Brownian motions. Based these, the likelihood ratio (LR) test statistic for cointegration rank is shown to be a functional of the standard Brownian motion and normal vector, asymptotically. As far as we know, our test is new in the literature. The critical values of the LR test are simulated via the Monte Carlo method. The performance of this test in finite samples is examined through Monte Carlo experiments. We apply our approach to an empirical example of three interest rates.

AMS subject classifications: 62M10, 37M10, 91B84

Key words: Vector AR model, cointegration, full rank estimation, vector GARCH process, partially nonstationary, reduced rank estimation.

*Corresponding author. *Email address:* maling@ust.hk (S. Ling)

1 Introduction

Since Granger [17] and Engle and Granger [13], the cointegrating time series has been a leading topic in the literature of economics. Numerous economic models, such as consumption function, purchasing power parity, money demand function, hedging ratio of spot and futures exchange rates, and yield curves of different terms of maturities, have been shown to have the cointigrating structure. The partially nonstationary Vector AR model or cointegrating time series models without GARCH effect have been extensively discussed over the past decades, see for example, Phillips and Durlauf [37] and Stock and Watson [43] in early years. Recently, Wang and Phillips [46] proposed a specification test for nonlinear non-stationary models. Kristensen and Rahbek [22] analyzed estimators and tests for a general class of VEC models that allows for asymmetric and non-linear error correction. Wang [45] established a martingale limit theorem for non-linear cointegration systems. Cavaliere *et al.* [9] considered bootstrap tests on the cointegration rank in vector AR models. Liang *et al.* [28] investigated local linear estimation of a nonparametric cointegration model. Cavaliere *et al.* [8] investigated a number of methods for estimating the cointegration rank in integrated vector AR systems with unknown AR order. Cai *et al.* [7] studied a new class of bivariate threshold cointegration models. Johansen and Nielsen [20] studied nonstationary cointegration in the fractionally cointegrated VAR model. Lin *et al.* [29] considered a double-nonlinear cointegration. She and Ling [40] studied a heavy-tailed VEC model. A recent overview on times series cointegration was given by Johansen [19].

Economic and financial time series often exhibit time-varying variances, called ARCH-type volatilities. Since Engle [11] and Bollerslve [4] proposed the ARCH/GARCH models, this kind of time series models have been extensively studied and applied in financial markets, see a nice review in Francq and Zakoian [14]. Ling *et al.* [31–34], Seo [39] established the asymptotic theory of the quasi-maximum likelihood estimator (QMLE) of unit root with the GARCH errors. Li *et al.* [27] investigated vector time series that exhibit both cointegration and time-varying variances. Li and Li [26] studied least absolute deviation estimation for unit root processes with GARCH errors. Chan and Zhang [10] provided an inference procedure for unit-root models with infinite variance GARCH errors. Lange [23] and Lange *et al.* [24] studied estimation and asymptotic inference for the AR-ARCH model. Shinki and Zhang [41] established asymptotic theory for fractionally integrated asymmetric power ARCH models. Zhang and Ling [48] established the asymptotic inference for AR models with heavy-tailed G-GARCH noises. Zhang *et al.* [47] studied an AR(1) model with ARCH(1) errors.