Journal of Computational Mathematics
Vol.42, No.1, 2024, 134–155.

http://www.global-sci.org/jcm
doi:10.4208/jcm.2209-m2021-0358

ALTERNATING OPTIMIZATION METHOD FOR ISOGEOMETRIC TOPOLOGY OPTIMIZATION WITH STRESS CONSTRAINTS*

Xiaoya Zhai[1)]
School of Mathematical Sciences, University of Science and Technology of China, Hefei 230071, China
Email: xiaoya93@mail.ustc.edu.cn

Abstract

Topology optimization (TO) has developed rapidly recently. However, topology optimization with stress constraints still faces many challenges due to its highly non-linear properties which will cause inefficient computation, iterative oscillation, and convergence guarantee problems. At the same time, isogeometric analysis (IGA) is accepted by more and more researchers, and it has become one important tool in the field of topology optimization because of its high fidelity. In this paper, we focus on topology optimization with stress constraints based on isogeometric analysis to improve computation efficiency and stability. A new hybrid solver combining the alternating direction method of multipliers and the method of moving asymptotes (ADMM-MMA) is proposed to solve this problem. We first generate an initial feasible point by alternating direction method of multipliers (ADMM) in virtue of the rapid initial descent property. After that, we adopt the method of moving asymptotes (MMA) to get the final results. Several benchmark examples are used to verify the proposed method, and the results show its feasibility and effectiveness.

Mathematics subject classification: 49J20, 65J15, 65N30.
Key words: Isogeometric topology optimization, Stress constraints, The ADMM-MMA solver.

1. Introduction

Topology optimization (TO) is a mathematical method that finds the optimal material layout by a given design domain under a set of loads, and boundary conditions [1]. It can be classified into two types according to the properties of different optimized objects, that are, topology optimization of continuum structures and discrete structures. Topology optimization of continuum structures is a hot topic nowadays. It can be applied to various fields, such as aerospace, bio-engineering, architectural design and so on.

Bendsoe and Kikuchi [2] first introduced the conception of the topology optimization of continuum structures and put forward a homogenization method that generated optimal topologies. Thereafter, a series of works came out. At present, there are many practical and effective numerical methods of topology optimization of continuum structures such as homogenization

* Received December 18, 2021 / Revised version received May 6, 2022 / Accepted September 13, 2022 / Published online March 14, 2023 /
1) Corresponding author

method [2], the solid isotropic material with penalization (SIMP) method [3, 4], the level set method [5], the moving morphable components (MMC) method [6, 7] and the moving morphable void (MMV) method [8, 9]. It is very difficult to solve topology optimization of continuum structures by the analytical method due to its nonlinearity. Most current methods adopt the numerical algorithm during calculation, which is based on finite element analysis (FEA). However, low-order shape functions often lead to numerical instabilities, slow convergence and iterative oscillation. Moreover, the gap and barrier between geometric representation and finite element analysis (FEA) still exist. The isogeometric analysis (IGA) offers us the opportunity for integrating computer-aided design (CAD) and computer-aided engineering (CAE), which was proposed by Huges *et al.* [10]. Recently, isogeometric analysis has already been applied to the field of topology optimization, which is called isogeometric topology optimization (ITO) because of its higher accuracy than traditional FEA. Kim *et al.* [11] extended IGA to the topology optimization by trimming the spline. Kumar *et al.* [12] used B-spline elements to represent the density function for topology optimization. They applied the implicit boundary method to boundary conditions and loads. Finally, they made a comparison with traditional elements and the results indicated that B-spline elements naturally tend to suppress shape irregularities and stability. Hassani *et al.* [13] proposed an isogeometric approach to topology optimization by optimality criteria. The results are independent of the number of discrediting control points and checkerboard free. Qian [14] represented the density distribution by B-splines over a rectangular domain, and this representation is compact in storage and does not require neighboring element information. Lin *et al.* [15] developed a unified strategy to simultaneously insert inclusions or holes of regular shape as well as the material to affect optimal topologies of solids. Wang *et al.* [16] introduced isogeometric topology optimization for periodic lattice materials to improve computational accuracy and efficiency and get faster convergence.

In practical engineering applications, the optimization problem with stress constraints is very important so that we can not ignore it. In 1996, Yand and Chen [17] studied stress-based topology optimization. They summarized the major difficulties of solving this problem. The first one is numerous constraints because of the local quantity of stress. This property will result in a huge computational burden of both optimization and sensitivity analysis. The second one is highly nonlinear with design variables. Later, Bendsoe and Sigmund [18] added one challenging issue, that is, the stress singularity phenomenon. The stress singularity arises in the density-based optimization approaches due to the discontinuity of local stress constraints when topology design variables tend to the critical values. It may prevent optimization algorithms from finding the true optimal material distribution [19]. We summarize three difficulties in topology optimization with stress constraints, which are as follows:

- Numerous stress constraints due to the local quantity of stress.
- Highly nonlinear properties.
- Stress singularity issues.

Later, researchers put forward different optimization models to solve the above three problems. Firstly, to avoid the huge computational burden created by local quality, the local stress constraints are often replaced with some stress aggregation formulation such as the P-norm function [20] and the Kreisselmeier Steinhauser (K-S) function [21]. However, instability will occur in the calculation process through the approximation of these two methods. The key is to make a trade-off between computational accuracy and efficiency. There are some works to