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IMPULSE NOISE REMOVAL BY L1 WEIGHTED NUCLEAR NORM MINIMIZATION*

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Abstract

In recent years, the nuclear norm minimization (NNM) as a convex relaxation of the rank minimization has attracted great research interest. By assigning different weights to singular values, the weighted nuclear norm minimization (WNNM) has been utilized in many applications. However, most of the work on WNNM is combined with the l^2-data-fidelity term, which is under additive Gaussian noise assumption. In this paper, we introduce the L1-WNNM model, which incorporates the l^1-data-fidelity term and the regularization from WNNM. We apply the alternating direction method of multipliers (ADMM) to solve the non-convex minimization problem in this model. We exploit the low rank prior on the patch matrices extracted based on the image non-local self-similarity and apply the L1-WNNM model on patch matrices to restore the image corrupted by impulse noise. Numerical results show that our method can effectively remove impulse noise.

Mathematics subject classification: 68U10, 94A08, 90C26, 15A03, 46N10.
Key words: Image denoising, Weighted nuclear norm minimization, l^1-data-fidelity term, Low rank analysis, Impulse noise.

1. Introduction

With the rapid development of technologies in image processing, many effective image denoising methods have been proposed based on the low rank matrix approximation (LRMA)

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that aims to restore a low rank matrix from its noisy or incomplete observation, e.g., in [1–3]. Generally, LRMA methods can be sorted into two categories: the nuclear norm minimization (NNM) methods, see [4–7], and the low rank matrix factorization (LRMF) methods, see [1,2,8]. In this paper, we focus on the first type. The NNM methods aim to seek a low rank solution by minimizing the nuclear norm and the work in [3] shows that many NNM-based problems can be solved via the nuclear norm proximal (NNP) that is defined as

$$\widehat{X} = \arg\min_{X\in\mathbb{R}^{m\times n}} \frac{1}{2}\|X-Y\|_F^2 + \lambda\|X\|_*, \tag{1.1}$$

where $Y \in \mathbb{R}^{m\times n}$ denotes the given observation, $\|\cdot\|_F$ denotes the Frobenius norm, $\|X\|_* = \sum_{i=1}^{l}\sigma_i(X)$ is the nuclear norm of X with $\sigma_i(X)$ as the i-th largest singular value of X, $l = \min(m,n)$, and $\lambda > 0$ is the regularization parameter. According to the work in [9], $\widehat{X}$ defined in (1.1) has a closed form, which can be obtained by using a soft-thresholding operation on the singular values of the observation matrix Y, that is,

$$\widehat{X} = \operatorname{prox}_{\lambda\|\cdot\|_*}(Y) = U\mathcal{D}_\lambda(\Sigma)V^T,$$

where $Y = U\Sigma V^T$ denotes the singular value decomposition (SVD) of Y, U and V are, respectively, $m\times l$ and $n\times l$ matrices with orthonormal columns, Σ is an $l\times l$ diagonal matrix with the main diagonal $[\sigma_1(Y),\sigma_2(Y),\cdots,\sigma_l(Y)]^T$, and $\mathcal{D}_\lambda : \mathbb{R}^{m\times n}\to\mathbb{R}^{m\times n}$ is an operator which applies the soft-thresholding on each element with parameter λ. Since all elements in Σ are non-negative, we have

$$\left(\mathcal{D}_\lambda(\Sigma)\right)_{i,i} = \max\left(\Sigma_{i,i}-\lambda, 0\right).$$

The main limitation of NNM methods is that all singular values are weighted equally, which may not be reasonable in some applications. As an example in image denoising, larger singular values are usually associated with the major image patterns and textures, while smaller singular values are usually associated with random noise. Thus, when we use NNM as regularization, the larger singular values should be weighted less in order to preserve major data components, while the smaller singular values should be weighted more in order to remove noise. In [10, 11] the weight nuclear norm $\|\cdot\|_{\omega,*}$ was proposed, which is defined as follows:

$$\|X\|_{\omega,*} = \sum_{i=1}^{l}\omega_i\sigma_i(X),$$

where $\omega = [\omega_1,\omega_2,\dots,\omega_l]^T$ with $\omega_i \geq 0$ for all $i = 1,\dots,l$ includes all weights. Combined with the l^2-data-fidelity term, a weighted nuclear norm minimization (WNNM) model was further proposed as follows:

$$\widehat{X} = \arg\min_{X\in\mathbb{R}^{m\times n}} \frac{1}{2}\|X-Y\|_F^2 + \|X\|_{\omega,*}. \tag{1.2}$$

The minimization problem defined in (1.2) is also called the weighted nuclear norm proximal (WNNP) problem. Its solution is a low rank approximation to Y and can be obtained efficiently as shown in [10, 11].

For the the l^2-data-fidelity term used in (1.2), it potentially assumes that the noise in Y is additive white Gaussian noise. However, in many applications different data-fidelity terms are considered to remove non-Gaussian noise [12–15]. For example, the l^1-data-fidelity term is usually used to remove impulse noise like the salt-and-pepper noise and the Laplace noise [12, 16–19]. In this paper, we combine the weighted nuclear norm with the l^1-data-fidelity term