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A Coiflet Wavelet Homotopy Technique for Nonlinear PDEs: Application to the Extreme Bending of Orthotropic Plate with Forced Boundary Constraints

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Abstract. A generalized homotopy-based Coiflet-type wavelet method for solving strongly nonlinear PDEs with nonhomogeneous edges is proposed. Based on the improvement of boundary difference order by Taylor expansion, the accuracy in wavelet approximation is largely improved and the accumulated error on boundary is successfully suppressed in application. A unified high-precision wavelet approximation scheme is formulated for inhomogeneous boundaries involved in generalized Neumann, Robin and Cauchy types, which overcomes the shortcomings of accuracy loss in homogenizing process by variable substitution. Large deflection bending analysis of orthotropic plate with forced boundary moments and rotations on nonlinear foundation is used as an example to illustrate the wavelet approach, while the obtained solutions for lateral deflection at both smally and largely deformed stage have been validated compared to the published results in good accuracy. Compared to the other homotopy-based approach, the wavelet scheme possesses good efficiency in transforming the differential operations into algebraic ones by converting the differential operators into iterative matrices, while nonhomogeneous boundary is directly approached dispensing with homogenization. The auxiliary linear operator determined by linear component of original governing equation demonstrates excellent approaching precision and the convergence can be ensured by iterative approach.

AMS subject classifications: 65M10, 78A48

Key words: Wavelet method, higher-order interpolating continuation, homotopy analysis method, geometric nonlinearity, orthotropic plate.

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1 Introduction

To obtain high-precision solution for nonlinear differential equations with inhomogeneous boundaries has been a critical issue in quantitative analysis of science and engineering, which is of great significance in developing effective approaches. Many numerical techniques have been proposed and can be classified into global and local techniques, with the former involving the derivatives of all points in the whole discrete domain, such as Fourier and Chebyshev spectral method [1], Discontinuous Galerkin [2], spectral element methods [3], spectral volume and difference method [4], while the local strategies obtain derivatives in terms of adjacent element, such as Finite Difference Method [5], Finite Element Method [6], Finite Volume Method [7], Boundary Element Method [8].

Homotopy Analysis Method (HAM) [9] has been an analytical powerful technique for dealing with strongly nonlinear problems, due to its freedom in selection of basis by leveraging the convergence properties in developing new numerical schemes. Von Gorder [10] has combined the Fourier method and the HAM to solve the large deflection of thin Kármán plate based on orthogonally sinusoidal basis in good agreement with exact solutions. Mosta et al. [11, 12] have formulated Spectral Homotopy Analysis Method (SHAM) by introducing Chebyshev and Legendre basis in the framework, which successfully overcome the limitations of initial guess and prove the convergence in Sobolev Spaces. Cullen and Clarke [13] have constructed Gegenbauer orthogonal basis expanding Chebyshev polynomials by Schmidt orthogonalization and proposed a fast and highly accurate Gegenbauer Homotopy Analysis Method, with the iterative matrix converted into sparse banded one up to machine precision, while the matrices of collocation points based on Chebyshev differential operators occupy large computational memory resource.

As a bright pearl of modern functional mathematics, wavelet [14, 15] has been an efficient tool in solving partial differential equations, due to its significant superiority on localized analysis. Early research on wavelet can be dated back to an orthogonal compactly supported Haar wavelet [16], which has been subsquently developed by many investigators [17–20]. Sweldens et al. [21, 22] have constructed a flexible wavelet lifting scheme [23], which are not necessarily the translates and dilates of one fixed function dependent of Fourier transformation. Dohono et al. [24–27] have applied a series of directional wavelets to study the characteristics of higher dimensional space introduced into multi-scale geometric analysis, such as Curvelets, Wedgelets, Ridgelets, Contourlets. Similar to early research all independent in various fields, present research of wavelet has not been formed a relatively unified framework and still in the process of exploration.

Many wavelet numerical methods [28–30] have been developed in solving differential equation, firstly studied by Qian and Weiss [31], which are in turn broadly categorised into single scale wavelet and adaptive methods, with the former indicating applying scaling function directly as basis in traditional methods, such as wavelet Galerkin method [32], wavelet collocation method [33], wavelet finite element method [34], closed wavelet method [35], wavelet multi-resolution interpolation Galerkin method [36, 37],