Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2023-0018

Vol. **34**, No. 3, pp. 837-868
September 2023

Multilevel Constrained Pressure-Temperature Residual Preconditioners for Large-Scale Non-Isothermal Reservoir Simulation via Restricted Additive Schwarz Algorithms

Mei Zhang[1], Haijian Yang[2,1,*], Chao Yang[3] and Shuyu Sun[4]

[1] *School of Mathematics, Hunan University, Changsha, Hunan, 410082, P.R. China.*
[2] *Shenzhen Research Institute, Hunan University, Shenzhen, 518000, PR China.*
[3] *School of Mathematical Sciences, Peking University, Beijing, 100871, P.R. China.*
[4] *Computational Transport Phenomena Laboratory, Physical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal, 23955-6900, Saudi Arabia.*

Received 13 January 2023; Accepted (in revised version) 17 June 2023

Abstract. The industry-standard constrained pressure residual (CPR) algorithm is often able to effectively improve the robustness behavior and the convergence speed of linear iterations for isothermal reservoir simulation. In this paper, we present and study an improved extension of CPR to the constrained pressure-temperature residual (CPTR) version for non-isothermal reservoir problems in heterogeneous porous media. In the proposed preconditioner, the corresponding approximations for the inverse of matrices are computed under a domain decomposition framework by using the restricted additive Schwarz (RAS) algorithm, to equally deal with the coupled thermal-pressure-saturation reservoir system and highly exploit the parallelism of supercomputer platforms. Moreover, we introduce and develop a family of multilevel CPTR preconditioners with suitable coarse grid corrections, to further improve the applicability of this two-stage preconditioner for large-scale computation. Numerical results for strong heterogeneous flow problems show that the new approach can dramatically improve the convergence of linear iterations, and demonstrate the superiority of CPTR over the commonly used RAS preconditioners. The parallel scalability of the non-isothermal reservoir simulator is also studied versus a supercomputer with tens of thousands of processors.

AMS subject classifications: 65F08, 49M15, 65M55, 68W10, 76S05, 90B05

Key words: Thermal reservoir simulation, porous media, preconditioning, restricted additive Schwarz algorithm, strong and weak scalability.

---

*Corresponding author. *Email addresses:* zhangmei8@hnu.edu.cn (M. Zhang), haijianyang@hnu.edu.cn (H. Yang), chao_yang@pku.edu.cn (C. Yang), shuyu.sun@kaust.edu.sa (S. Sun)

1 Introduction

Thermal reservoir simulation of flow in porous media is widely applied in many reservoir applications from heavy oil recovery, and geothermal energy to carbon sequestration by fully coupling the temperature and the other flow variables [9, 24, 36]. The extended non-isothermal flow accounting for the fluid properties is affected by the temperature including the density, viscosity, capillary pressure effects, relative permeability functions, and so on. As a result, non-isothermal reservoir simulation inherits the original difficulties for solving the large-scale multiphase flow problems in porous media, including the high heterogeneity of the media, the multiple variations of the density and viscosity, the complexity of the geometry, etc. Moreover, the additional complexity of thermal simulation comes from the dominant mechanism of heat conduction through the rock by the persistent interaction between the fluid and temperature, which poses extremely challenging to the computational methodology for predictive numerical simulation. The focus of this study is on this critical need in particular by designing a parallel and highly-efficient reservoir simulator, which is based on the constrained pressure residual (CPR) method and a restricted additive Schwarz (RAS) based solver for enabling large-scale computation on supercomputers.

The CPR method, first proposed by Wallis [39, 40] in the early 1980s, is a class of widely used preconditioning techniques in the community of reservoir simulation. This physics-splitting preconditioner decouples and solves the pressure and other variables respectively to deal with the corresponding blocks based on their different properties. As an industry-standard preconditioner in reservoir simulation, many CPR-based preconditioning techniques are also developed and have been increasingly applied by many researchers to a large number of reservoir applications [5, 12, 15, 17, 41, 42]. However, the extension of the CPR works to non-isothermal reservoir problems is not straightforward, because heat conduction through rock and fluids can dominate the mechanism of the fluid flow, and the second stage of CPR struggles to capture the heat diffusion, which results in the convergence degradation of CPR [11,30]. To improve the flexibility and capability of CPR, the references [29,30] present a family of constrained pressure-temperature residual (CPTR) algorithms for solving the non-isothermal reservoir problems, where a block preconditioner based on a new Schur-complement expression is introduced for the pressure-temperature system. In another relevant work, a family of multi-stage CPTR preconditioners is presented in [11] for solving the thermal–compositional–reactive flow in porous media, where a Schur-complement decomposition is applied to extract a temperature subsystem and an additional multigrid preconditioning stage is proposed to improve the treatment of the energy equation. The numerical results from some challenging two-phase dead-oil or thermal–compositional–reactive cases show the reduced sensitivity to the thermal reservoir regime and improve the convergence behavior of reservoir simulators.

On the other hand, the restricted additive Schwarz (RAS) algorithm, proposed by Cai [7, 8] in 1999, is based on the overlapping domain decomposition technique and