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Impacts of Down-Up Hill Segment on the Threshold of Shock Formation of Ring Road Vehicular Flow

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Abstract. The study of impacts of down-up hill road segment on the density threshold of traffic shock formation in ring road vehicular flow is helpful to the deep understanding of sags' bottleneck effect. Sags are freeway segments along which the gradient increases gradually in the traffic direction. The main aim of this paper is to seek the density threshold of shock formation of vehicular flow in ring road with down-up hill segment, because down-up hill roadway segment is a source to cause capacity reduction that is an attractive topic in vehicular traffic science. To seek the density threshold numerically, a viscoelastic continuum model [1] is extended and used. To solve the model equations, a fifth-order weighted essentially non-oscillatory scheme for spatial discretization, and a 3rd order Runge-Kutta scheme for time partial derivative term are used. Validation by existing observation data and the Navier-Stokes like model [2] extended as EZM is done before conducting extensive numerical simulations. For ring road vehicular flow with three separated down-up hill segments, it is found that the density threshold of shock formation decreases monotonically with the relative difference of free flow speed, this variation can be simply fitted by a third order polynomial.

AMS subject classifications: 35Q80, 76A02

Key words: Down-up hill road segment, viscoelastic continuum model, sags' bottleneck effect, density threshold, WENO5 scheme.

1 Introduction

To investigate the effects of road infrastructure condition on traffic flow dynamics, a research background has been given in [1], from which the existing results of several stud-

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ies reported in [3–8] can be sought.

However, less work has been done for the down-up hill segment effects on the density threshold of traffic shock formation in ring road vehicular flow from macroscopic points of view. Therefore, by assuming that traffic sound speed of traffic flow at a state over second critical point is just equal to the second critical sound speed, this paper extends the macroscopic viscoelastic continuum model (VEM) reported in [1] and uses the extended model to predict the density threshold of traffic shock formation and travel time numerically. The ring road is composed of three separated down-up hill segments and horizontal road segment, as shown in Fig. 1. As the down segment is linked immediately by a length-identical uphill, thus we call the composed as the down-up hill segment or the sag. Indeed, due to the role of gravitational acceleration, there is a shorter braking distance for vehicles on uphill segment, but a longer braking distance for vehicles on downhill segment, with the vehicles on horizontal segment having a braking distance between the two. Generally, braking distance is a function of free flow speed [9]. Consequently, there should have three relevant free flow speeds on the three road segments. Therefore, for the ring road with down-up hill segment, the traffic fundamental diagram is section dependent, as shown in Fig. 2.

To validate the VEM, the existing observation data are used for comparison of traffic flow states, and the Navier-Stokes like model of Zhang [2] extended as EZM [10] is adopted to provide the counterpart results for validation. It is assumed that the total length of the ring road is 120km, and the single downhill or uphill length is 1km.

The main aim of this paper is to seek the density threshold of shock formation of vehicular flow in ring road with down-up hill segment. Hence, on the basis of the VEM, a fifth-order weighted essentially non-oscillatory scheme (WENO5) [11,12] and a third-order Runge-Kutta scheme (RK3) [13,14] are used to build a simulation platform. Model validation is done before conducting extensive numerical simulations of ring road vehicular flow to explore the density threshold and its variation trend. We will introduce the VEM and numerical method just before method of travel time prediction, then discuss the results, and finally give the conclusions.

2 Viscoelastic traffic flow model

The viscoelastic traffic flow model (VEM) [1] uses traffic pressure derived by assuming the explicit algebraic form of traffic sound speed and the definition of the sound speed in classical mechanics, rather than governed by a partial differential equation in the gas-kinetic-based model [15,16]. Assuming traffic density ρ is normalized by traffic jam density ρ_m, when velocity scale is v_0, the traffic flow rate q has a unit of $\rho_m v_0$. For length scale l_0, time scale is $t_0 = l_0/v_0$. Taking normalized traffic density ρ and normalized traffic flow rate q as mandatory variables, neglecting ramp effects, and defining traffic elasticity by