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On Inhibition of the Rayleigh-Taylor Instability by a Horizontal Magnetic Field in 2D Non-Resistive MHD Fluids: The Viscous Case

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Abstract. We investigate whether the inhibition phenomenon of the Rayleigh-Taylor (RT) instability by a horizontal magnetic field can be mathematically verified for a non-resistive viscous magnetohydrodynamic (MHD) fluid in a two-dimensional (2D) horizontal slab domain. This phenomenon was mathematically analyzed by Wang (J. Math. Phys., 53:073701, 2012) for stratified MHD fluids in the linearized case. To our best knowledge, the mathematical verification of this inhibition phenomenon in the nonlinear case still remains open. In this paper, we prove such inhibition phenomenon for the (nonlinear) inhomogeneous, incompressible, viscous case with Navier (slip) boundary condition. More precisely, we show that there is a critical number of the field strength $m_{\rm C}$, such that if the strength $|m|$ of a horizontal magnetic field is bigger than $m_{\rm C}$, then the small perturbation solution around the magnetic RT equilibrium state is algebraically stable in time. Moreover, we also provide a nonlinear instability result when $|m|\in[0,m_{\rm C})$.

AMS subject classifications: 35Q60, 35B10, 76E25

Key words: Non-resistive viscous MHD fluids, Rayleigh-Taylor instability, algebraic decay-in-time, stability/instability threshold.

1 Introduction

The equilibrium of a heavier fluid on top of a lighter one, subject to gravity, is unstable. In fact, small disturbances acting on the equilibrium will grow and lead to the release of

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potential energy, as the heavier fluid moves down under gravity, and the lighter one is displaced upwards. This phenomenon was first studied by Rayleigh [38] and then Taylor [41], is called therefore the Rayleigh-Taylor (RT) instability. In the last decades, the RT instability has been extensively investigated from mathematical, physical and numerical aspects, see [3, 5, 6, 27, 42] for examples. It has been also widely analyzed how physical factors, such as elasticity [30, 34], rotation [2, 3], internal surface tension [14, 18, 45], magnetic fields [21, 28, 29] and so on, influence the dynamics of the RT instability.

In this paper we are interested in the inhibition phenomenon of the RT instability by magnetic fields. This topic goes back to the theoretical work of Kruskal and Schwarzchild [35]. They analyzed the effect of an (impressed) horizontal magnetic field upon the growth of the RT instability in a horizontally periodic motion of a completely ionized plasma with zero resistance in three dimensions in 1954, and pointed out that the curvature of the magnetic lines can influence the development of instability, but can not inhibit the growth of the RT instability. The inhibition of the RT instability by a vertical magnetic field was first verified for inhomogeneous, incompressible, non-resistive magnetohydrodynamic (MHD) fluids in three dimensions by Hide [3, 17]. In 2012 Wang also noticed that a horizontal magnetic field can inhibit the RT instability in a non-resistive MHD fluid in two dimensions [43]. Later, Jiang and Jiang further found that impressed magnetic fields always inhibit the RT instability, if a non-slip velocity boundary condition is imposed in the direction of the magnetic fields [20]. Such boundary condition is called the "fixed condition" to emphasize that the two endpoints (on the upper and lower boundaries of a horizonal slab domain, respectively) of magnetic lines are fixed.

All results mentioned above are based on the linearized non-resistive MHD equations. Thanks to the multi-layers method developed in the well-posedness theory of surface wave problems [15], recently the inhibition phenomenon of the RT instability by magnetic fields has been rigorously proved based on the nonlinear non-resistive viscous MHD equations under the fixed condition, for example, Wang verified the inhibition phenomenon by a non-horizontal magnetic field in stratified incompressible viscous MHD flows in a 2D/3D slab domain [44]. Moreover, he also proved that a horizontal magnetic field can not inhibit the RT instability for the horizontally periodic motion in 3D [44], but can inhibit the linear RT instability in 2D [43]. Similar results can be obtained for other magnetic inhibition phenomena, see [22] for the Parker instability and [25] for the thermal instability. The aforementioned nonlinear stability/instability results on the non-resistive viscous magnetic RT problem can be summarized in the Table 1.

Table 1: Can an impressed horizontal/vertical magnetic field inhibit the nonlinear RT instability in non-resistive viscous MHD flows in a slab domain?

	Horizontal	Vertical
2D	Not clear	Yes
3D	No	Yes

Jiang and Jiang [24] further established a so-called magnetic inhibition theory in viscous non-resistive MHD fluids, which reveals the physical effect of the fixed condition