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# Numerical Algorithm with Fourth-Order Spatial Accuracy for Solving the Time-Fractional Dual-Phase-Lagging Nanoscale Heat Conduction Equation

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**Abstract.** Nanoscale heat transfer cannot be described by the classical Fourier law due to the very small dimension, and therefore, analyzing heat transfer in nanoscale is of crucial importance for the design and operation of nano-devices and the optimization of thermal processing of nano-materials. Recently, time-fractional dual-phase-lagging (DPL) equations with temperature jump boundary conditions have showed promising for analyzing the heat conduction in nanoscale. This article proposes a numerical algorithm with high spatial accuracy for solving the time-fractional dual-phase-lagging nano-heat conduction equation with temperature jump boundary conditions. To this end, we first develop a fourth-order accurate and unconditionally stable compact finite difference scheme for solving this time-fractional DPL model. We then present a fast numerical solver based on the divide-and-conquer strategy for the obtained finite difference scheme in order to reduce the huge computational work and storage. Finally, the algorithm is tested by two examples to verify the accuracy of the scheme and computational speed. And we apply the numerical algorithm for predicting the temperature rise in a nano-scale silicon thin film. Numerical results confirm that the present difference scheme provides $\min\{2-\alpha, 2-\beta\}$ order accuracy in time and fourth-order accuracy in space, which coincides with the theoretical analysis. Results indicate that the mentioned time-fractional DPL model could be a tool for investigating the thermal analysis in a simple nanoscale semiconductor silicon device by choosing the suitable fractional order of Caputo derivative and the parameters in the model.

**AMS subject classifications**: 35R11, 65M06, 65M12, 80A20

**Key words**: Nanoscale heat transfer, fractional dual-phase-lagging model, finite difference scheme, stability, convergence.

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## 1. Introduction

For the problem related to self-heating in micro-electronic device or for thermal dynamics involving ultrashort-pulsed laser heating, the dual-phase-lagging (DPL) constitutive relation, i.e.,

$$\mathbf{q}(\mathbf{r},t)+\tau_q\frac{\partial \mathbf{q}}{\partial t}(\mathbf{r},t)=-\kappa\left\{\nabla \mathrm{T}(\mathbf{r},t)+\tau_T\frac{\partial}{\partial t}\nabla \mathrm{T}(\mathbf{r},t)\right\} \tag{1.1}$$

is one of the best descriptions of such heat transfer [2,6,7,9,13–15,18,21,23,27,28,30, 34,35]. Here, $\mathbf{r}$ stands for a material point, $t$ is the time, $\kappa$ is the thermal conductivity of the material, $\mathbf{q}$ is the heat flux, $\mathrm{T}$ is the temperature, $\nabla$ is the gradient operator, $\tau_q, \tau_T$ are the phase lags of the heat flux $q$ and temperature gradient $\nabla\mathrm{T}$, respectively. The DPL constitutive relation considers the effect of finite relaxation time using both heat flux and temperature phase lags where the former is caused by structural interactions such as phonon scattering and the latter is interpreted as the relaxation time due to fast-transient effects of thermal inertia [28]. In other words, although the DPL constitutive relation is a macroscopic formulation, it views the interactions on the microscopic level as retarding sources causing a delayed response at the macroscopic scale. Hence, using the DPL constitutive relation to analyze the thermal behavior with the micro/nano structural effect should be convenient to practicing engineers.

In the past decades, due to the memorability and dependency of the fractional calculus, it has found many practical applications in different fields of science and engineering [3, 8, 16, 24, 29]. Recently, some fractional models have been successfully applied to simulate the heat and thermal transfer in non-uniform porous medium, viscoelastic materials, dynamic electro-magnetic fields, etc. Sherief *et al.* [22] proposed a class of fractional non-Fourier law with the concept of the fractional derivative. Results show a good agreement with experimental data when using fractional derivatives for description of viscoelastic materials. Youssef [32] investigated another form of fractional non-Fourier law with the concept of the fractional integral. Povstenko [20] further studied a time-fractional Cattaneo-type equations and formulated corresponding theories of thermal stresses. Yu *et al.* [33] developed a fractional order generalized electro-magneto-thermo-elasticity theory for anisotropic and linearly electro-magneto-thermo-elastic media by introducing the dynamic electro-magnetic fields. And numerical results show the fractional order has great effect on the response when material is imposed a sudden heating. Mishra [17] developed a fractional single-phase-lagging heat conduction model by applying fractional Taylor series formula and investigated the effect of different parameters on temperature solution. Yang and Chen [31] employed the fractional single-phase lag heat conduction to predict both temperature distribution around cracks and the transient fracture behaviors considering the viscoelastic material properties, and found that the fractional single-phase lag heat conduction theory may avoid the negative temperature predicted by the hyperbolic heat conduction disappears. Chi *et al.* [1] built a time fractional heat conduction equation to study the heat transfer process in coaled methane adsorption.