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Absorbing Interface Conditions for the Simulation of Wave Propagation on Non-Uniform Meshes

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Abstract. We proposed absorbing interface conditions for the simulation of linear wave propagation on non-uniform meshes. Based on the superposition principle of second-order linear wave equations, we decompose the interface condition problem into two subproblems around the interface: for the first one the conventional artificial absorbing boundary conditions is applied, while for the second one, the local analytic solutions can be derived. The proposed interface conditions permit a two-way transmission of low-frequency waves across mesh interfaces which can be supported by both coarse and fine meshes, and perform a one-way absorption of high-frequency waves which can only be supported by fine meshes when they travel from fine mesh regions to coarse ones. Numerical examples are presented to illustrate the efficiency of the proposed absorbing interface conditions.

AMS subject classifications: 35K10, 65N06

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1. Introduction

In many numerical simulations of physical phenomenons such as the structural response of materials and wave propagation in medium, spurious wave motions are among the most basic problems. The problem initially arises in the traditional numer-

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ical methods like finite element method (FEM). Documentation in this aspect can be traced back to the 17-th century by Brillouin in [8]. Bažant has pointed out the reflection phenomenon of elastic wave in passing across the interface between different scale meshes [3,4,9]. Furthermore, as an important method dealing with artificial interface [6,7,11,12,17,25,26], the local time step method also suffers from the same problem [23].

Another related source of the spurious reflection is the interface of regions with different modeling scale [2,18,24]. For example, as one of the most important work in the area of multi-scale modelling and computation, the heterogeneous multiscale method (HMM) developed by E *et al.* [13,21,23,28] successfully couples the molecular dynamics model and the continuum model [16,20]. This method is based on the domain decomposition method, which combines the finite element method and the Cauchy-Born rule. In order to eliminate the spurious reflection around interface, Li *et al.* proposed a variational boundary condition as the continuum-atomistic interface condition [23]. Recently, they further developed an extended Galerkin projection method to deal with this spurious reflection generated at the interface [22], which extends the projection space and the equation at the interface to decrease reflection. Other work includes the bridging scale method presented by Liu and his coworkers [27,30], in which they have used a projection process to couple atomistic and continuum simulations by bridging scale decomposition.

Replica time integration (RTI) [29, 33], on the other hand, also plays a significant role recently in this field. It builds up an interface condition that allows two-way transmission for high-frequency waves. The two-way transmission afforded by RTI is accomplished by representing the state of the coarse regions through replica ensembles, consisting of collections of identical copies of the coarse regions. Similar replica ensembles are commonly used in statistical physics to describe the equilibrium thermodynamic properties of systems such as spin glasses. In RTI, the replicas within each ensemble run on their slow time steps and are out-of-phase concerning each other by one fast time step. With the aid of this device, high-frequency signals in the fine regions of the model can be effectively transmitted into coarse regions as ensembles of replica waves with a minimum of spurious internal reflections at the interface. Conversely, ensembles of replica waves in the coarse regions of the model are transmitted as high-frequency signals into the fine regions. The RTI requires that the coarse meshes have to be an integer multiple of the fine meshes.

In this article, we would focus on the elimination of spurious reflections generated at interface by waves passing from fine mesh to coarse mesh. The main aim is to design an interface condition that allows a two-way transmission of low-frequency waves and performs a one-way absorption of high-frequency waves that can only be supported by fine meshes. Based on the finite-difference stencils represented by RTI, we choose to reconstruct the interface scheme by applying the absorbing boundary conditions proposed in the landmark work [14] by Engquist and Majda, instead of using replica ensembles to afford a two way transmission at the price of repeated calculation for each replica with slow time steps. More precisely, with the help of superposition principle for