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GLOBAL EXISTENCE AND LONG-TIME BEHAVIOR FOR THE STRONG SOLUTIONS IN H^2 TO THE 3D COMPRESSIBLE NEMATIC LIQUID CRYSTAL FLOWS*

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Abstract

In this paper, we investigate the global existence and long time behavior of strong solutions for compressible nematic liquid crystal flows in three-dimensional whole space. The global existence of strong solutions is obtained by the standard energy method under the condition that the initial data are close to the constant equilibrium state in H^2-framework. If the initial datas in L^1-norm are finite additionally, the optimal time decay rates of strong solutions are established. With the help of Fourier splitting method, one also establishes optimal time decay rates for the higher order spatial derivatives of director.

Keywords compressible nematic liquid crystal flows; global solution; Green function; long-time behavior

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1 Introduction

In this paper, we investigate the motion of compressible nematic liquid crystal flows, which are governed by the following simplified version of the Ericksen-Leslie equations

$$\begin{cases}\rho_t + \mathrm{div}(\rho u) = 0,\\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) - \mu\Delta u - (\mu+\nu)\nabla \mathrm{div} u + \nabla P(\rho) = -\gamma \nabla d \cdot \Delta d,\\ d_t + u\cdot\nabla d = \theta(\Delta d + |\nabla d|^2 d),\end{cases} \tag{1.1}$$

where ρ, u and d stand for the density, velocity and macroscopic average of the nematic liquid crystal orientation field respectively. The pressure $P(\rho)$ is a smooth

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function in a neighborhood of 1 with $P'(1) = 1$. The constants μ and ν are shear viscosity and the bulk viscosity coefficients of the fluid respectively, that satisfy the physical assumptions

$$\mu > 0, \quad 2\mu + 3\nu \geq 0.$$

The positive constants γ and θ present the competition between the kinetic energy and the potential energy, and the microscopic elastic relaxation time for the molecular orientation field, respectively. For simplicity, we set the constants γ and θ to be 1. The symbol $\otimes$ denotes the Kronecker tensor product such that $u \otimes u = (u_i u_j)_{1 \leq i,j \leq 3}$. To complete system (1.1), the initial data are given by

$$(\rho, u, d)(x, t)|_{t=0} = (\rho_0(x), u_0(x), d_0(x)). \tag{1.2}$$

Furthermore, as the space variable tends to infinity, we assume

$$\lim_{|x| \to \infty} (\rho_0 - 1, u_0, d_0 - w_0)(x) = 0, \tag{1.3}$$

where w_0 is a fixed unit constant vector. The system is a coupling between the compressible Navier-Stokes equations and a transported heat flow of harmonic maps into S^2. Generally speaking, we can obtain any better results for system (1.1) than those for the compressible Navier-Stokes equations.

The hydrodynamic theory of liquid crystals in the nematic case has been established by Ericksen [1] and Leslie [2] during the period of 1958 through 1968. Since then, the mathematical theory is still progressing and the study of the full Ericksen-Leslie model presents relevant mathematical difficulties. The pioneering work comes from [3-6]. For example, Lin and Liu [5] obtained the global weak and smooth solutions for the Ginzburg-Landau approximation to relax the nonlinear constraint $d \in S^2$. They also discussed the uniqueness and some stability properties of the system. Later, the decay rates for this approximate system were given by Wu [7] in a bounded domain. On the other hand, Dai et al. [8], Dai and Schonbek [9] established the time decay rates for the Cauchy problem respectively. More precisely, Dai and Schonbek [9] obtained the global existence of solutions in the Sobolev space $H^N(\mathbb{R}^3) \times H^{N+1}(\mathbb{R}^3)$ $(N \geq 1)$ only requiring the smallness of $\|u_0\|_{H^1}^2 + \|d_0 - w_0\|_{H^2}^2$, where w_0 is a fixed unit constant vector. If the initial data in L^1-norm are finitely additionally, they also established the following time decay rates

$$\|\nabla^k u(t)\|_{L^2} + \|\nabla^k (d - w_0)(t)\|_{L^2} \leq C(1+t)^{-\frac{3+2k}{4}},$$

for $k = 0, 1, 2, \cdots, N$. Recently, Liu and Zhang [10], for the density-dependent model, obtained the global weak solutions in dimension three with the initial density $\rho_0 \in L^2$, which was improved by Jiang and Tan [11] for the case $\rho_0 \in L^\gamma$ $(\gamma > \frac{3}{2})$. Under the constraint $d \in S^2$, Wen and Ding [12] established the local existence of