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BIFURCATIONS AND NEW EXACT TRAVELLING WAVE SOLUTIONS OF THE COUPLED NONLINEAR SCHRÖDINGER-KdV EQUATIONS*

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Abstract

By using the method of dynamical system, the exact travelling wave solutions of the coupled nonlinear Schrödinger-KdV equations are studied. Based on this method, all phase portraits of the system in the parametric space are given. All possible bounded travelling wave solutions such as solitary wave solutions and periodic travelling wave solutions are obtained. With the aid of Maple software, the numerical simulations are conducted for solitary wave solutions and periodic travelling wave solutions to the coupled nonlinear Schrödinger-KdV equations. The results show that the presented findings improve the related previous conclusions.

Keywords dynamical system method; coupled nonlinear Schrödinger-KdV equations; solitary wave solution; periodic travelling wave solution; numerical simulation

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1 Introduction

In recent years, the investigation of the exact travelling wave solutions to nonlinear wave equations plays an important role in nonlinear science, since the exact travelling wave solutions can provide much physical information and more insight of the physical and mathematical aspects of the problem and then lead to further applications. Several effective methods for obtaining exact travelling wave solutions of nonlinear wave equations, such as (G'/G)-expansion method [1], the theta function method [2], Darboux and Backlund transform [3], tanh-coth method [4], sine/cosine

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method [5], Jacobi elliptic function expansion method [6], the homogeneous balance method [7], the symmetry method [8], functional variable method [9] have been developed. Among them, the dynamical system method is one of these effective methods which has been applied to many nonlinear wave equations [10,11].

In this paper, we consider the following coupled nonlinear Schrödinger-KdV equations [12]

$$\begin{cases} iu_t + u_{xx} = uv, \\ v_t + \alpha v v_x + \beta v_{xxx} = (|u|^2)_x, \end{cases} \tag{1.1}$$

where α, β are real parameters. u is a complex function and v is a real function. The study of coupled nonlinear Schrödinger-KdV equations has attracted extensive interest in physics and mathematics. Many numerical methods have been used to solve numerically the single nonlinear Schrödinger and the single KdV equation using finite element and finite difference methods [13-16]. Analytical solutions of the coupled nonlinear Schrödinger-KdV equations using different methods were given in [17-19]. Here, we shall use the dynamical system method to seek exact travelling wave solutions of (1.1).

In order to find travelling wave solutions of (1.1), we assume that

$$u(x,t) = \phi(\xi)\mathrm{e}^{i\eta}, \quad v(x,t) = \psi(\xi), \quad \xi = kx - ct, \quad \eta = px + lt, \tag{1.2}$$

where k, c, p and l are travelling wave parameters.

Substituting (1.2) into the first equation of (1.1), canceling $\mathrm{e}^{i\eta}$ and separating the real and imaginary parts, we have

$$\begin{cases} \phi'(2kp - c) = 0, \\ k^2\phi'' - (p^2 + l)\phi - \phi\psi = 0. \end{cases} \tag{1.3}$$

Obviously, from (1.3), we know that if $\phi' = 0$, then (1.1) has a trivial solution. Otherwise, (1.3) must be satisfied

$$2kp - c = 0. \tag{1.4}$$

Substituting (1.2) into the second equation of (1.1), and integraling once (integral constant is zero), we have

$$\beta k^3\psi'' - c\psi + \frac{\alpha k}{2}\psi^2 - k\phi^2 = 0. \tag{1.5}$$

Therefore, (1.1) is reduced to

$$\begin{cases} 2kp - c = 0, \\ k^2\phi'' - (p^2 + l)\phi - \phi\psi = 0, \\ \beta k^3\psi'' - c\psi + \frac{\alpha k}{2}\psi^2 - k\phi^2 = 0. \end{cases} \tag{1.6}$$

It is very difficult to solve this equations by some ordinary methods, so we consider the special transformation in subtle ways