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EFFECTS OF A TOXICANT ON A SINGLE-SPECIES POPULATION WITH PARTIAL POLLUTION TOLERANCE IN A POLLUTED ENVIRONMENT*†

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Abstract

We study a model for the long-term behavior of a single-species population with some degree of pollution tolerance in a polluted environment. The model consists of three ordinary differential equations: one for the population density, one for the amount of toxicant inside the living organisms, and one for the amount of toxicant in the environment. We derive sufficient conditions for the persistence and the extinction of the population depending on the exogenous input rate of the toxicant into the environment and the level of pollution tolerance of the organisms. Numerical simulations are carried out to illustrate our main results.

Keywords single-species population; pollution tolerance; toxicant; persistence; extinction; long-term behavior

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1 Model Formulation

Hallam *et al.* [1-3] proposed a deterministic model to describe the effects of a toxicant in the environment on a single species population. In their model they assumed that the environment is constant and that the population growth rate depends linearly on the concentration of the toxicant present inside the organisms. Later modifications of the model (see [4-18] and references therein) introduced nonlinearities into the population growth rate as a function of the toxicant concentration as well as fluctuating parameters representing a variable environment. In this paper we present a version of the model with a variable environment as well as a partial pollution tolerance to the toxicant. Our main aim is to study the effect of different

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levels of pollution tolerance on the long-term survival of the population in an environment with a varying input rate of the toxicant into the environment.

We consider a spatially well-mixed population of identical individuals. The model contains three state variables: the population biomass $x(t)$ at time t, the concentration $C_o(t)$ of the toxicant present in the organisms (that is, amount of toxicant per biomass), and the concentration $C_e(t)$ of the toxicant in the environment. The rate of change of the biomass at time t is described by the following differential equation:

$$\dot{x}(t) = x(t)(b-d) - (\alpha C_o(t) + \beta C_e(t))\Big(x(t) - \frac{\gamma x(t)}{1+ax(t)}\Big) - cx^2(t), \tag{1.1}$$

where b and d represent the birth and death rates in the absence of the toxicant respectively; α and β are the population responses to the toxicant in the organisms and in the environment respectively; c is the competition strength among the individuals of population; and the term $(\alpha C_o(t)+\beta C_e(t))\big(x(t)-\frac{\gamma x(t)}{1+ax(t)}\big)$ describes losses due to not being possessed pollution tolerance; the item $\frac{\gamma x(t)}{1+ax(t)}$ denotes the Holling type II functional response where γ represents the pollution tolerance; $b, d, \alpha, \beta, \gamma$, a and c are all positive constants.

The total amount of toxicant present in the organisms is $x(t)C_o(t)$. The rate of change of $x(t)C_o(t)$ depends on uptake and loss and is given by the following equation:

$$\begin{aligned}\frac{\mathrm{d}(x(t)C_o(t))}{\mathrm{d}t} &= kx(t)C_e(t) - gx(t)C_o(t) - mx(t)C_o(t) - dx(t)C_o(t) \\ &\quad -(\alpha C_o(t) + \beta C_e(t))\Big(x(t) - \frac{\gamma x(t)}{1+ax(t)}\Big)C_o(t) - cx^2(t)C_o(t), \end{aligned}\tag{1.2}$$

where $kx(t)C_e(t)$ is the toxicant uptake rate from the environment by the population. The toxicant is lost from the organisms due to a number of causes: $gx(t)C_o(t)$ describes loss due to egestion; $mx(t)C_o(t)$ describes loss due to metabolic processes; $dx(t)C_o(t)$ denotes loss due to death of population; $(\alpha C_o(t) + \beta C_e(t))\big(x(t) - \frac{\gamma x(t)}{1+ax(t)}\big)C_o(t)$ represents loss due to partial pollution tolerance inside and outside the organisms; $cx^2(t)C_o(t)$ represents loss due to intra-competition of population. Substituting (1.1) into (1.2) yields

$$\dot{C}_o(t) = kC_e(t) - (g+m+b)C_o(t). \tag{1.3}$$

Let m_o be the mass of per capita and m_e be the total mass of the medium in the environment. Then $m_eC_e(t)$ is the total amount of the toxicant in the environment at time t, denoting it by $y_e(t)$. The losses of the toxicant from the environment include the part taken up by population at a rate $km_ox(t)C_e(t)$ as well as purification of the environment at a rate $hy_e(t)$. The increases of the toxicant in the environment con-