Ann. of Appl. Math.
33:4(2017), 340-352

EXISTENCE OF SOLUTIONS FOR NONLOCAL BOUNDARY VALUE PROBLEM OF FRACTIONAL DIFFERENTIAL EQUATIONS ON THE INFINITE INTERVAL*

Abdellatif Ghendir Aoun†
(*Ecole Normale Supérieure, 16050 Kouba, Algiers, Algeria*)

Abstract

In this paper, we study a fractional differential equation

$$^{c}D_{0^+}^{\alpha}u(t) + f(t, u(t)) = 0, \quad t \in (0, +\infty)$$

satisfying the boundary conditions:

$$u'(0) = 0, \quad \lim_{t\to+\infty} {}^{c}D_{0^+}^{\alpha-1}u(t) = g(u),$$

where $1 < \alpha \leqslant 2$, $^{c}D_{0^+}^{\alpha}$ is the standard Caputo fractional derivative of order α. The main tools used in the paper is a contraction principle in the Banach space and the fixed point theorem due to D. O'Regan. Under a compactness criterion, the existence of solutions are established.

Keywords boundary value problem; fractional differential equation; infinite interval; nonlocal condition; fixed point theorem

2000 Mathematics Subject Classification 34A08; 34B40

1 Introduction

In this paper, we consider the following boundary value problem (BVP for short)

$$\begin{cases} ^{c}D_{0^+}^{\alpha}u(t) + f(t, u(t)) = 0, \quad t \in (0, +\infty), \\ u'(0) = 0, \quad \lim_{t\to+\infty} {}^{c}D_{0^+}^{\alpha-1}u(t) = g(u), \end{cases} \tag{1.1}$$

where $1 < \alpha \leqslant 2$ and $f : [0, +\infty) \times \mathbb{R} \to \mathbb{R}$, $g : X \to \mathbb{R}$ are the given functions such that X is a suitable Banach space.

Fractional differential equations arise in many engineering and scientific disciplines as the mathematical models of systems and processes in the fields of physics, chemistry, electrical circuits, biology, and so on, and involves derivatives of fractional order. Fractional derivatives provide an excellent tool for the description of memory

*Manuscript received April 13, 2017; Revised August 14, 2017
†Corresponding author. E-mail: ghendirmaths@gmail.com

and hereditary properties of various materials and processes. This is the main advantage of fractional differential equations in comparison with classical integer-order models. Further, the concept of nonlocal boundary conditions has been introduced to extend the study of classical boundary value problems. This notion is more precise for describing nature phenomena than the classical notion because additional information is taken into account. For the importance of nonlocal conditions in different fields, in the paper, we let $g(u) = \sum\limits_{i=1}^{p} c_i u(\xi_i)$, where $c_1, c_2, \cdots, c_p$ are given constants with $p \in \mathbb{N}^*$, and $0 < \xi_1 < \xi_2 < \ldots < \xi_p < +\infty$ as an application of the results we will get.

The problem studied in this paper is very well motivated in relationship with several previous contributions. The main difficulty in treating this class of the fractional differential equations is the possible lack of compactness due to the infinite interval, besides the boundary condition which prevents from proving compactness conditions. In order to overcome these difficulties, the authors use a special Banach space which can establish some similar inequalities as finite interval. These better properties can be guarantee that the operator is completely continuous.

The mathematical investigation of such problems has been the subject of several research works during the last years (see, e.g., [1-5,7,9-16]).

In [1], Arara, Benchohra, Hamidi and Nieto discussed the existence of bounded solutions, using Schauder's fixed point theorem, of the following problem on an unbounded domain:

$$\begin{cases} {}^cD_{0^+}^{\alpha} y(t) = f(t, y(t)) = 0, \quad t \in J := [0, +\infty), \\ u(0) = y_0, \quad y \text{ is bounded on } J, \end{cases}$$

where $1 < \alpha \leqslant 2$ and $y_0 \in \mathbb{R}$.

In [5], Liang and Zhang considered the m-point BVP of fractional differential equation on unbounded interval:

$$\begin{cases} D_{0^+}^{\alpha} u(t) + a(t) f(t, u(t)) = 0, \quad t \in (0, +\infty), \\ u(0) = 0, \quad u'(0) = 0, \quad D_{0^+}^{\alpha-1} u(+\infty) = \sum\limits_{i=1}^{m-2} \beta_i u(\xi_i), \end{cases}$$

where $2 < \alpha \leqslant 3$. Using a fixed point theorem for operators on a cone of a Banach space, sufficient conditions for the existence of multiple positive solutions were established.

Su and Zhang [9] discussed the existence of unbounded solutions of the following BVP using Schauder's fixed point theorem:

$$\begin{cases} D_{0^+}^{\alpha} u(t) + f(t, u(t), D_{0^+}^{\alpha-1} u(t)) = 0, \quad t \in (0, +\infty), \\ u(0) = 0, \quad u'(0) = 0, \quad D_{0^+}^{\alpha-1} u(\infty) = u_{\infty}, \quad u_{\infty} \in \mathbb{R}, \end{cases}$$