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SUMUDU TRANSFORM IN BICOMPLEX SPACE AND ITS APPLICATIONS*

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Abstract

In this paper, we define Sumudu transform with convergence conditions in bicomplex space. Also, we derive some of its basic properties and its inverse. Applications of bicomplex Sumudu transform are illustrated to find the solution of differential equation of bicomplex-valued functions and find the solution for Cartesian transverse electric magnetic (TEM) waves in homogeneous space.

Keywords Sumudu transform; bicomplex numbers; bicomplex functions and bicomplex Laplace transform

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1 Introduction

In literature, various integral transforms have been widely used in physics and engineering mathematics. In the sequence of these integral transforms, Watugala [26] defined Sumudu transform and applied it to find the solution of ordinary differential equations in control engineering problems.

Over the set of functions

$$\mathcal{A} = \big\{ f(t): \text{ there exists an } M,\ \tau_j > 0,\ |f(t)| < M\mathrm{e}^{\frac{|t|}{\tau_j}},\ \text{if } t \in (-1)^j[0,\infty),\ j = 1,2 \big\}, \tag{1}$$

the Sumudu transform is defined by the formula

$$\mathcal{S}[f(t);s] = \frac{1}{s}\int_0^\infty \mathrm{e}^{-\frac{t}{s}} f(t)\mathrm{d}t, \quad s \in (\tau_1, \tau_2). \tag{2}$$

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Sumudu transform has scale and unit preserving properties. In [11], Belgacem et al. discussed fundamental properties of Sumudu transform and showed that Sumudu transform is a theoretical dual of Laplace transform. Also, it is used to solve an integral production-depreciation problem. In [10], Belgacem and Karaballi generalized Sumudu differentiations, integrations and convolution theorems existing in the previous literatures. Also they generalized Sumudu shifting theorems and introduced recurrence formulas of the transform.

In [27], Zhang developed an algorithm based on Sumudu transform which can be implemented in computer algebra systems like Maple and used to solve differential equations. In [17], Hussian and Belgacem obtained the solution of Maxwell's differential equations for transient excitation functions propagating in a lossy conducting medium by using Sumudu transform in time domain.

In [9], Belgacem found the electric field solutions of Maxwell's equations, pertaining to transient electromagnetic planner, (TEMP), waves propagating in lossy media, through Sumudu transform. In [13], Eltayeb et al. discussed the Sumudu transform on a space of distributions. In [19, 20], Kilicman and Gadain produced some properties and relationship between double Laplace and double Sumudu transform and also, used the double Sumudu transform to solve wave equation in one dimension having singularity at initial conditions.

In [18], Kilicman et al. discussed the existence of double Sumudu transform with convergence conditions and applied it to find the solution of linear ordinary differential equations with constant coefficients. In [6], Al-Omari and Belgacem investigated certain class of quaternions and Sumudu transform. Motivated by the work of Al-Omari et al., we make efforts to extend the Sumudu transform to bicomplex variable.

2 Bicomplex Number

In 1892, Segre Corrado [25] defined bicomplex numbers as

$$C_2 = \{\xi : \xi = x_0 + i_1 x_1 + i_2 x_2 + j x_3 |\ x_0, x_1, x_2, x_3 \in C_0\},$$

or

$$C_2 = \{\xi : \xi = z_1 + i_2 z_2 |\ z_1, z_2 \in C_1\},$$

where i_1 and i_2 are imaginary units such that $i_1^2 = i_2^2 = -1$, $i_1 i_2 = i_2 i_1 = j$, $j^2 = 1$ and C_0, C_1 and C_2 are sets of real numbers, complex numbers and bicomplex numbers, respectively. The set of bicomplex numbers is a commutative ring with unit and zero divisors. Hence, contrary to quaternions, bicomplex numbers are commutative with some non-invertible elements situated on the null cone.