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PARTIAL REGULARITY RESULT OF SUPERQUADRATIC ELLIPTIC SYSTEMS WITH DINI CONTINUOUS COEFFICIENTS*†

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Abstract

We consider the partial regularity for weak solutions to superquadratic elliptic systems with controllable growth condition, under the assumption of Dini continuous coefficients. The proof relies upon an iteration scheme of a decay estimate for a new type of excess functional. To establish the decay estimate, we use the technique of $\mathcal{A}$-harmonic approximation and obtain a general criterion for a weak solution to be regular in the neighborhood of a given point. In particular, the proof yields directly the optimal Hölder exponent for the derivative of the weak solutions on the regular set.

Keywords superquadratic elliptic systems; controllable growth condition; $\mathcal{A}$-harmonic approximation; optimal partial regularity

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1 Introduction

The purpose of this papers is to study the partial regularity of the weak solutions $u \in W^{1,m}(\Omega, R^N)$, to second order superquadratic elliptic systems in divergence form

$$-\mathrm{div}A(x,u,Du) = B(x,u,Du) \quad \text{in } \Omega, \tag{1.1}$$

where Ω is a bounded domain in $R^n (n \geq 2)$, the structure function $A : \Omega \times R^N \times R^{nN} \longrightarrow R^{nN}$ satisfies some standard ellipticity and growth conditions, see (H1) and (H2) below. The inhomogeneity $B(x, \xi, p)$ satisfies the controllable growth condition, see (C) below. N is an integer with $N > 1$, u and B take values in R^N. We make these notion precisely for the specific structure conditions and the assumptions in the next section.

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We first give a short overview of the partial regularity idea. The idea of partial regularity runs through the last century. In 1968, De Giorgi demonstrated in [5] that, in contrast to equations, one cannot in general expect that weak solutions to (1.1) to be regular everywhere on the domain even under reasonable assumptions on A and B of (1.1). We can only expect a partial regularity result for general nonlinear systems, which means that solutions are only known to be regular outside a singular set of Lebesgue measure zero. Results of this type have been established for elliptic systems with quadratic case by Giaquinta and Modica in [11], see also Ivert [18]. The blow-up technique was earlier applied in the setting of elliptic systems by Giusti and Miranda [15]. Recently, a more elementary proof of regularity of minimizers of elliptic integrals in Geometric Measure Theory was proposed by Duzaar and Steffen [10] on the basis of the $\mathcal{A}$-harmonic approximation method, which is inspired again by the original methods of De Giorgi [4,5] and later used by Simon [19,20]. The method was successfully applied to study the quadratic elliptic systems in [8], where Duzaar and Grotowski gave a simplified proof of their result without $L^p - L^2$ estimates for the derivative Du of u. The superquadratic case was later on dealt by Hamburger [17] and Chen and Tan in [3], and the sub-quadratic case goes back to Beck [1]. Until now, the technique of harmonic approximation has been developed further and adapted to various settings in the regularity theory, cf. [9] for a survey on the numerous applications of harmonic type approximation lemmas.

All results on partial regularity stated above are concerned with the case of coefficients being Hölder continuous with respect to $(x;u)$, Duzaar and Gastel [7] weakened the assumptions on A with Dini continuity that is

$$(1+|p|)^{-1}|A(x,\xi,p)-A(\overline{x},\overline{\xi},p)| \le K(|\xi|)\mu(|x-\overline{x}|+|\xi-\overline{\xi}|),$$

and proved a partial regularity result under the natural growth condition for the case of $m=2$.

In the superquadratic case, where $u \in W^{1,m}(\Omega, R^N)$, $m>2$ and the coefficients A satisfy the Dini continuity condition and inhomogeneity $B(x,\xi,p)$ satisfies the controllable growth condition, only few partial regularity results are known. By using the $\mathcal{A}$-harmonic approximation technique, we extend the results in [7] to the case of $m>2$, different from the related results of the above mentioned papers. We assume for the continuity of A with respect to the variables (x,ξ) that

$$(1+|p|)^{-\frac{2}{m}}|A(x,\xi,p)-A(\overline{x},\overline{\xi},p)| \le K(|\xi|)\mu\big((|x-\overline{x}|^m+|\xi-\overline{\xi}|^m)^{\frac{\beta}{m}}\big). \tag{1.2}$$

For most direct proofs, the crucial step of improving an inequality of Caccioppoli or reverse-Poincaré type, in order to be able to gain decay estimates on