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DYNAMICS OF A PREDATOR-PREY REACTION-DIFFUSION SYSTEM WITH NON-MONOTONIC FUNCTIONAL RESPONSE FUNCTION*†

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Abstract

In this article, a two-species predator-prey reaction-diffusion system with Holling type-IV functional response and subject to the homogeneous Neumann boundary condition is regarded. In the absence of the spatial diffusion, the local asymptotic stability, the instability and the existence of Hopf bifurcation of the positive equilibria of the corresponding local system are analyzed in detail by means of the basic theory for dynamical systems. As well, the effect of the spatial diffusion on the stability of the positive equilibria is considered by using the linearized method and analyzing in detail the distribution of roots in the complex plane of the associated eigenvalue problem. In order to verify the obtained theoretical predictions, some examples and numerical simulations are also included by applying the numerical methods to solve the ordinary and partial differential equations.

Keywords reaction-diffusion system; predator-prey system; asymptotic stability; Hopf bifurcation

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1 Introduction

Since Lotka [5] and Volterra [13] proposed the classical Lotka-Volterra predator-prey model, the dynamics between predator and their prey have drawn great attention of many researchers including mathematicians and biologists. The functional response function included in the classical Lotka-Volterra predator-prey model is linear through the origin and therefore is unbounded. In the research of the predator-prey systems, however, people recognized that the functional response of predator, in

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some case, is nonlinear and bounded. Thus a more reasonable predator-prey system should include a nonlinear and bounded functional response function. Let u and v denote the population densities of prey and predator species at time t, respectively. Then the dynamics between predator and prey species should be described by the following model [3, 7, 8, 14]

$$\begin{cases} \dot{u} = ug(u, K) - vp(u), \\ \dot{v} = v(-D + q(u)), \\ u(0) = u_0 \geq 0, \quad v(0) = v_0 \geq 0, \end{cases} \tag{1.1}$$

where K is the most existent capacity of prey and D is the death rate of predator. The function $g(u, K)$ represents the growth rate of prey in the absence of predator and is assumed to satisfy the following conditions when $u \geq 0$ and $K > 0$:

$$g(K, K) = 0, \quad g(0, K) > 0, \quad \lim_{K\to\infty} g(0, K) < \infty,$$

$$g_u(u, K) < 0, \quad g_K(u, K) \geq 0, \quad g_{uK}(u, K) > 0, \quad \lim_{K\to\infty} g_u(u, K) = 0.$$

The form most often used for $g(u, K)$ is the so-called Logistic growth of the form $g(u, K) = 1 - \frac{u}{K}$. The predator response function $p(u)$, in general, is a continuously differentiable function defined on $[0, \infty)$ and satisfies

$$p(0) = 0, \quad p(u) > 0 \quad \text{for } u > 0.$$

To describe the group defense of prey species, it is assumed that there exists a constant $M > 0$ such that

$$p'(u) \begin{cases} > 0, & 0 \leq u < M, \\ < 0, & u > M. \end{cases}$$

The function $q(u)$ represents the rate of conversion of the captured prey to predator and, generally, in the models of Gauss type, $q(u)$ is taken as $cp(u)$ where c is a positive constant.

According to the above properties of the response function $p(u)$, Andrews [1] proposed the following Monod-Haldane (also called Holling type-IV) response function:

$$p(u) = \frac{mu}{a + bu + u^2}, \tag{1.2}$$

where $m > 0$ represents the maximal growth rate of the species, $a > 0$ is the half-saturation coefficient and b is a positive constant. On the basis of the response function (1.2), Sokol and Howell [12] in 1981 suggested the simplified form of (1.2), namely,

$$p(u) = \frac{mu}{a + u^2}. \tag{1.3}$$