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DYNAMICAL ANALYSIS OF A SINGLE-SPECIES POPULATION MODEL WITH MIGRATIONS AND HARVEST BETWEEN PATCHES*†

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Abstract

A single-species population model with migrations and harvest between the protected patch and the unprotected patch is formulated and investigated in this paper. We study the local stability and the global stability of the equilibria. The research points out, under some suitable conditions, the single-species population model admits a unique positive equilibrium, which is globally asymptotically stable. We also derive that the trivial solution is globally asymptotically stable when the harvesting rate exceeds the threshold. Further, we discuss the practical effects of the protection zones and the harvest. The main results indicate that the protective zones indeed eliminate the extinction of the species under some cases, and the theoretical threshold of harvest to the practical management of the endangered species is provided as well. To end this contribution and to check the validity of the main results, numerical simulations are separately carried out to illustrate these results.

Keywords protection zone; migrations; harvesting; extinction
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1 Introduction

The environmental pollution and the unregulated harvesting/hunting threaten the life of wild animals and the biological diversity. In order to prevent the biological resources from destruction and to protect the ecological environment, the human beings therefore have taken various measures to deal with these situations. The

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most and popular measure is to establish the protection zone to the endangered species in the recent decades.

The species with the unregulated harvest from human beings have been paid more attention by the researchers, and the related improvements have been derived by various techniques. Within recent two decades, the models with protection zones have been investigated by the researchers and scholars around the world, where the single-species population models were focused on in [1-5], the predator-prey models were studied in [6-9] and the competitive model was considered in [10]. Among these results, the stability and optimal harvest of the corresponding models were presented in [1-3], and the diffusion or migration-dependent models were extensively discussed in [1-10]. We would like to mention the work by Fan and Wang [1], in which they considered a model on harvested population with diffusion migration:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = rx(t)\Big(1 - \dfrac{x(t)}{K}\Big) - d(x(t) - y(t)) - Ex(t), \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = ry(t)\Big(1 - \dfrac{y(t)}{K}\Big) + d(x(t) - y(t)), \end{cases} \tag{1}$$

where $x(t)$ is the density of the species in the natural environment and $y(t)$ is the density of the same species in the natural reserve at time t, respectively; r is the intrinsic growth rate; K is the carry capacity of the environment. The item $d(x(t) - y(t))$ represents the diffusion capacity that is the total biomass caused by the diffusion effect, where d is the proportional coefficient and is assumed to be proportional to the difference of the densities.

According to the reports from the statisticians and the official investigations by the forestry of China, for the endangered species, the density of most species in the natural environment (also called the unprotected patch, non-protection zone) is less than the density in the natural reserve (also called the protected patch, protection zone), such as the Tibetan antelopes, the Asian elephants and other rare species. However, food-dependent and/or temperature-dependent distributions of the species, the plenty of food source and health care are the main factors for the species to migrate from one patch to another. That is, the diffusion of the species is actually a double-direction process between two patches. We denote the immigration rate from the unprotected to protected patch by m_1, and the emigration rate from the protected to unprotected by m_2, instead of one-direction diffusion process of model (1). And we also assume that $m_1 \geq m_2$ throughout this paper.

When it comes to the financial benefit of the species, the hunters generally take high risk to harvest the individuals in the unprotected patch. Throughout this paper, we always assume that E_1 and E_2 are the hunting rates in the unprotected