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# The PAT Model of Population Dynamics

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**Abstract.** We introduce a population-age-time (PAT) model which describes the temporal evolution of the population distribution in age. The surprising result is that the qualitative nature of the population distribution dynamics is robust with respect to the birth rate and death rate distributions in age, and initial conditions. When the number of children born per woman is 2, the population distribution approaches an asymptotically steady state of a kink shape; thus the total population approaches a constant. When the number of children born per woman is greater than 2, the total population increases without bound; and when the number of children born per woman is less than 2, the total population decreases to zero.

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## 1 Introduction

In human history, technological advances are the main factors for human population increase, such as tool-making revolution, agricultural revolution and industrial revolution. Technological advances provide human with more food supply and medicine. More food supply is the main driver of population increase. Medicine prolongs human life span. Diseases such as plagues could cause human population to temporarily decrease. But since 1700, human population has been monotonically increasing

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due to technological advances. Since 1960s, due to the introduction of high yield grains, agricultural machineries, fertilizers, chemical pesticides, and better irrigation systems, human population has been increasing by 1 billion every 12 years, from 3 billion to 8 billion by 2023. Thus enrichment of food has increased human population dramatically. It seems that both the Cornucopian and the Malthusian views were realized [6, 7]. Human indeed dramatically advanced technology to provide abundant food supply to meet the demand of population growth according to Cornucopian view. Human population also dramatically increased with the abundant food supply according to Malthusian view. The question is whether or not we are heading to a new Malthusian catastrophe, i.e. some people are going to starve. Technologies may be advanced further to support more humans. But the earth resource is limited, and the human population cannot increase without bound on earth.

Human overpopulation not only can cause huge damage to earth resource and environment, but also has serious sustainability consequence. If there is a global food scarcity, huge famine can cause major population loss. According to World Wide Fund for Nature [9], the current human population is already exceeding its earth carrying capacity. On the other hand, estimating earth's carrying capacity for human is more difficult than for other animals due to the fact that human choices may play an important role [1]. In the long run, human population cannot continue to grow; there are clear human resource limits of food, energy and territory (individual human space) as discussed by von Hoerner [5]. The key moment is when human population reaches its maximum. The crucial question is: How will the human population change afterward? Will human population more or less stay at a stagnation population or decrease substantially? If human population decreases, is the decrease due to birth control, normal death or abnormal death? Birth control and normal death are hopeful for reducing human population from the example of China. Abnormal death corresponds to various kinds of disasters such as diseases, wars etc.. Von Hoerner also proposed the possibility of moving humans out of earth, i.e. stellar expansion [5]. But wars and diseases are more probable.

There were studies on human population dynamics from logistic point of view [8] and ecological perspectives [4]. Here we are focusing on the temporal evolution of the population distribution in age, and introduce the population-age-time model.

## 2 The population-age-time (PAT) model

Let $p(t,a)$ be the population of age $a$ (in year) at time $t$ (in year). One can think $p(t,a)$ as the spectrum of population in age, $a$=0,1,2,$\cdots$,$A$; $t$=0,1,2$\cdots$. We introduce