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Magnetoelastic Instability of a Long Graphene Nano-Ribbon Carrying Electric Current

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Abstract. This paper aims at investigating the resonance frequencies and stability of a long Graphene Nano-Ribbon (GNR) carrying electric current. The governing equation of motion is obtained based on the Euler-Bernoulli beam model along with Hamilton's principle. The transverse force distribution on the GNR due to the interaction of the electric current with its own magnetic field is determined by the Biot-Savart and Lorentz force laws. Using Galerkin's method, the governing equation is solved and the effect of current strength and dimensions of the GNR on the stability and resonance frequencies are investigated.

AMS subject classifications: 82D80, 37N15, 74H55

Key words: Graphene nano-ribbon, resonance frequency, current carrying, Hamilton's principle, Biot-Savart, Lorenz force.

1 Introduction

Recent progresses in nanotechnology have led to the development of nano-electro-mechanical systems (NEMS). Carbon nanostructures such as nanotubes, nanocones and graphene nanoribbons are widely used as nanosensores, nanomechanical resonators, nanoswitches robotic manipulators and magneto-elastic biosensors. The very high stiffness, low density, specific optical properties, high current carrying capability and having two-dimensional structure, have attracted the attention of scientists to GNRs [1–4]. Owing to these outstanding properties, graphene is an ideal material for the design and

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development of new NEMS for a variety of applications, including force, position and mass sensing [5–9].

The structural instability is one of the major problems encountered in flexible lightweight components of NEMS. The vibration and instability of a current-carrying elastic rods have been studied by some researchers [10–13]. Also, recently Chen and et al. [14] investigated the fabrication and electrical readout of monolayer Graphene resonators, and studied their response to changes in mass and temperature.

The aim of this study is to investigate the resonance frequencies and instability of a long GNR carrying electric current. The Lorentz force produced by the interaction of the current with its own magnetic field induces the transverse deflection of GNR. The GNR is modeled as an Euler-Bernoulli beam and the Galerkin method is applied to solve the governing equation of motion. Based on the obtained model, the variation of resonance frequencies and instability conditions of the GNRs of different dimensions are investigated.

2 Governing equations of motion

Fig. 1 shows a schematic of a GNR of flexural rigidity D, length l, width b and thickness h which carries electric current I. The transverse vibration of the GNR is described in the global xyz frame so that the x axis coincides with the neutral axis. The GNR is suspended across a valley between two metallic gates, and is bridge at both ends. Considering the GNR as an Euler-Bernoulli beam, the governing equation of transverse deflection can be derived using Hamilton's principle;

$$\delta H = \int_{t_1}^{t_1} \delta(K-U+W)dt = 0, \tag{2.1}$$

where K is the kinetic energy, U is the potential energy, and W is the work done by the self induced Lorentz force. The kinetic and potential energies of the beam are given by

$$K = \frac{1}{2}\int_0^l \rho A \dot{w}^2 dx, \tag{2.2a}$$

$$U = \frac{1}{2}\int_0^l D w''^2 dx, \tag{2.2b}$$

where ρA is mass of GNR per unit length and w is the transversal deflection. Also the prime and dot symbols denote the derivative with respect to x and time, respectively. The work done by a transverse force distribution f_y on the GNR is calculated as

$$W = \int_0^l f_y w dx. \tag{2.3}$$

The GNR can be modeled as a series of differential segments as shown in Fig. 2. The magnetic field due to the element dx_1 on the neutral axis at point x is obtained from the