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A Lifting Method for Krause's Consensus Model

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Abstract. In this work, we aim to show how to solve the continuous-time and continuous-space Krause model by using high-order finite difference (FD) schemes. Since the considered model admits solutions with δ-singularities, the FD method cannot be applied directly. To deal with the annoying δ-singularities, we propose to lift the solution space by introducing a splitting method, such that the δ-singularities in one spatial direction become step functions with discontinuities. Thus the traditional shock-capturing FD schemes can be applied directly. In particular, we focus on the two-dimensional case and apply a fifth-order weighted nonlinear compact scheme (WCNS) to illustrate the validity of the proposed method. Some technical details for implementation are also presented. Numerical results show that the proposed method can capture δ-singularities well, and the obtained number of delta peaks agrees with the theoretical prediction in the literature.

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1 Introduction

It is believed that the finite difference (FD) method cannot be applied directly for solving problems with δ-singularities appearing in the solutions. For problems with δ-singularities entering initial value conditions or source terms, one may regularize the Dirac delta function by using a nonsingular function (see [17] and references therein), and then apply the traditional FD method as usual. However, these regularizations may smear the solutions severely and lead to large errors in the approximation. In addition, this approach would encounter difficulty when δ-singularities arise in the solution as time goes on. Krause's model in continuous form is typically one of the models that δ-singularities may appear in the solution for long time, even for a smooth initial condition. Moreover, the number of delta peaks and the positions vary rapidly for different definitions of the velocity. Thus, it is hard to construct a stable FD scheme for this model.

Krause's model in discrete form has been widely used to analyze the dynamics of multiagent consensus, which has important applications in many disciplines, such as control theory [6] and information engineering [1]. In applications, people are often concerned about the case with a large number of agents in high dimensions. However, it is difficult to analyze the behavior of this case in a discrete framework. As a fundamental step, it is worth studying the dynamical models for continuous distributions of agents; see [2, 5, 18] for examples.

The so-called continuous-time and continuous-space Krause model presented in [5] reads as

$$\rho_t + \operatorname{div}(V\rho) = 0, \tag{1.1}$$

where ρ represents the density and $V = (u,v,w,\ldots)$ the velocity that depends on ρ in a nonlinear way

$$V(\boldsymbol{x},t) = \int_{B_R(\boldsymbol{x})} (\boldsymbol{y}-\boldsymbol{x})\rho(\boldsymbol{y},t)\,\mathrm{d}\boldsymbol{y}. \tag{1.2}$$

Here the integral domain $B_R(\boldsymbol{x})$ is the closed ball

$$B_R(\boldsymbol{x}) = \{\boldsymbol{y} \,|\, \|\boldsymbol{y}-\boldsymbol{x}\| \le R\}, \tag{1.3}$$

where $\boldsymbol{x}$ represents a spatial vector, the norm $\|\cdot\|$ can be the Euclidean norm or the maximum norm, and R is the radius of the ball. For simplicity of the computational domain, only the maximum norm is considered in this work.

Since δ-singularities may appear in the solution of Eq. (1.1), it is not surprised that the widely used FD method [7, 8, 15, 21] cannot be applied directly, especially for high-dimensional cases. Thus some authors paid their attention to the finite volume (FV) method [4, 5] and discontinuous Galerkin (DG) method [12, 19, 20],