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A General Cavitation Model for the Highly Nonlinear Mie-Grüneisen Equation of State

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Abstract. A general one-fluid cavitation model is proposed for a family of Mie-Grüneisen equations of state (EOS), which can provide a wide application of cavitation flows, such as liquid-vapour transformation and underwater explosion. An approximate Riemann problem and its approximate solver for the general cavitation model are developed. The approximate solver, which provides the interface pressure and normal velocity by an iterative method, is applied in computing the numerical flux at the phase interface for our compressible multi-medium flow simulation on Eulerian grids. Several numerical examples, including Riemann problems and underwater explosion applications, are presented to validate the cavitation model and the corresponding approximate solver.

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Key words: Multi-phase flow, one-fluid cavitation model, approximate Riemann solver, Mie-Grüneisen EOS, underwater explosion.

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1. Introduction

Fluid flow with cavitation occurs when the low pressure in the liquid reaches towards the limit of vapour pressure. One of the most famous cavitation is the flow generated in an underwater explosion near a structure and a free surface, due to collisions of the shock and expansive waves. The dominant difficulties in simulating such kinds of cavitating flows include dynamical phase creation, dynamical interface creation, and the treatment of cavitation evolution and collapse.

Various methods have already been developed for the purpose of cavitation simulation, including interface tracking methods and two-phase methods. The interface

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tracking method assumes that there is a clear and distinct interface between the liquid and vapour, which is determined via an iterative procedure [5, 7, 8]. While the two-phase method makes no attempt to track the liquid and vapour interface, but instead treats the flow as a mixture of two-phases with an averaging mixture density, which continuously varies between the liquid and vapour extremes [21–24].

In recent years, the two-phase method is becoming more and more popular partly because it is able to treat all the possible physics of cavitating flows. In this implementation, there are generally two different approaches, the two-fluid method and the one-fluid method. The two-fluid method assumes that both phases coexist inside a zone cell simultaneously and each phase follows its own governing equations respectively. This method is often used to simulate the compressible multiphase and multi-medium flow, in which the exchange of mass, momentum and energy are treated explicitly as transfer terms. Hence, two-fluid methods can easily calculate mass and energy exchange, heat transfer and even surface tension occurring at the cavitation interface [22,24,30]. There are several classical two-fluid models in literatures, based on different assumptions, such as seven-equation model, six-equation model, five-equation model, and so on. For more details, please see the review papers of Saurel [22] and Linga [18].

Different from the two-fluid method, the one-fluid method treats the two phases as a homogenous and barotropic mixture. Therefore, one set of differential equations similar to the single-phase flow expresses the whole fluid motion. The most important and difficult part of this approach is to define a proper equation of state for the mixture that covers the fluid phase changing from a liquid state to the vapor state. Because the flow parameters obtained are in the averaging sense for the one-fluid model, it is difficult for such to resolve the detailed physics or quantities related to phase transition. However, it is very easy to treat the dynamic formation, evolution and collapse of cavitation, and is more computationally efficient than the two-fluid method. So far, various one-fluid methods have been developed for the cavitation simulation in compressible flows. The cut-off model [1] is one of the prominent models, which is mainly used for cavitation simulations and, almost exclusively, for simulations of underwater explosions. If the flow pressure falls below a critical value, the pressure is set to a known value (usually saturation vapor pressure). In this model, the conservation law is violated, and the nature of hyperbolic systems of equations may change nonphysically due to the cut-off of pressure and density or zero sound velocity in a cavitation zone [19, 28]. The Schmidt model was developed by Schmidt *et al.* [25] for a high-pressure and high-velocity flow in a small nozzle, where the cavitation flow is assumed as a homogeneous and barotropic mixture of gas and liquid. The equation of state is obtained for the mixture by integrating a predefined sound velocity in the two-phase region. In fact, the Schmidt model can be employed for cases with small density ratio or cases in which the pressure jumps across cavitation boundaries do not exceed a certain limit ($\rho_g/\rho_l < 10^{-5}$). Liu *et al.* proposed an isentropic model [19] where the vapour component of the cavitation mixture is assumed to be homogeneous, compressible, and isentropic. The proposed equation of state is based on an iterative method to calculate the pressure in the cavitation region. The model is appropriate