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Large Deviations for Heavy-tailed Random Variables in Prospective-loss Process*

BAO ZHEN-HUA
(*School of Mathematics, Liaoning Normal University, Dalian, 116029*)

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Abstract: In this paper, we study the precise large deviations for the prospective-loss process with consistently varying tails. The obtained results improve some related known ones.

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1 Introduction

In this paper, we discuss the following prospective-loss process

$$W(t) = \sum_{k=1}^{N(t)} (X_k - (1+\delta)\mu), \tag{1.1}$$

where $\delta \geq 0$ is a constant, and $\{X_k,\ k \geq 1\}$ is a sequence of independent and identically distributed (i.i.d.) nonnegative random variables with distribution function (d.f.) $F = 1 - \overline{F}$ and finite mean μ, $\{N(t),\ t \geq 0\}$ denotes a nonnegative integer-valued process, which is independent of the sequence $\{X_k,\ k \geq 1\}$. We assume that

$$\lambda(t) = EN(t) < \infty, \qquad \forall t \geq 0$$

but

$$\lambda(t) \to \infty \qquad \text{as } t \to \infty.$$

We are interested in probabilities of large deviations under the assumption that F is heavy-tailed. A non-negative random variable X or its d.f. F concentrated on $[0, \infty)$ is said to be heavy-tailed if it has no any finite exponential moment. One of the most important heavy-tailed subclasses is the subexponential class $\mathcal{S}$. By definition, a d.f. F with support on $[0,\ \infty)$ belongs to $\mathcal{S}$ if

$$\lim_{x\to\infty} \frac{\overline{F^{n*}}(x)}{\overline{F}(x)} = n$$

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holds for any $n \geq 2$ (or, equivalently, for $n = 2$), where $\overline{F^{n*}}$ denotes the tail of n-fold convolution of F. There are some other important classes of heavy-tailed distributions which are intimately related to the class $\mathcal{S}$. A d.f. F is said to belong to the class $\mathcal{D}$ of dominately varying tails concentrated on $(-\infty, \infty)$ if

$$\limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} < \infty$$

holds for every $y \in (0,1)$ (or, equivalently, for some $y \in (0,1)$). A slightly smaller class is $\mathcal{C}$, which consists of all distributions with consistent variation in the sense that

$$\lim_{y\downarrow 1} \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1,$$

or equivalently,

$$\lim_{y\uparrow 1} \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1.$$

Another well-known heavy-tailed subclass is the so-called class ERV (extended regularly varying class). By definition, a distribution function F with support $[0, \infty)$ belongs to the class ERV$(-\alpha, -\beta)$ for $1 < \alpha \leq \beta < \infty$ if

$$y^{-\beta} \leq \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \leq \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \leq y^{-\alpha} \qquad \text{for any } y > 1.$$

It is well-known that the following inclusion relationships hold:

$$\text{ERV}(-\alpha, -\beta) \subset \mathcal{C} \subset \mathcal{D} \cap \mathcal{S}.$$

The precise large deviations for random sums $W(t)$ in (1.1) were studied by Ng *et al.*[1] under the following Assumptions on $N(t)$.

Assumption A As $t \to \infty$

$$\frac{N(t)}{\lambda(t)} \xrightarrow{P} 1.$$

Assumption B For any fixed small constant $\theta > 0$ and some small $\varepsilon > 0$,

$$EN^{\beta+\varepsilon}(t)\mathbf{1}_{(N(t)>(1+\theta)\lambda(t))} = O(\lambda(t)).$$

The main results derived in [1] are as follows.

Proposition 1.1 *Let $\{X_k, k \geq 1\}$ be a sequence of i.i.d. ERV$(-\alpha, -\beta)$ random variables with $1 < \alpha \leq \beta < \infty$. We assume that $\{X_k,\ k \geq 1\}$ is independent of the nonnegative and integer-valued process $\{N(t),\ t \geq 0\}$. Furthermore, we suppose that $N(t)$ satisfies Assumption A. Then, for any fixed $\gamma > 0$,*

$$P\Big(\sum_{k=1}^{N(t)} (X_k - (1+\delta)\mu) > x\Big) \sim \lambda(t)\overline{F}(x + \delta\lambda(t)\mu) \tag{1.2}$$

uniformly for $x \geq \gamma\lambda(t)$.

Proposition 1.2 *Let $\{X_k,\ k \geq 1\}$ be a sequence of i.i.d. ERV$(-\alpha, -\beta)$ random variables with $1 < \alpha \leq \beta < \infty$. We assume that $\{X_k,\ k \geq 1\}$ is independent of the nonnegative*