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Meshfree Approximation for Stochastic Optimal Control Problems

Hui Sun and Feng Bao*

Department of Mathematics, Florida State University, Tallahassee, Florida.

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Abstract. In this work, we study the gradient projection method for solving a class of stochastic control problems by using a mesh free approximation approach to implement spatial dimension approximation. Our main contribution is to extend the existing gradient projection method to moderate high-dimensional space. The moving least square method and the general radial basis function interpolation method are introduced as showcase methods to demonstrate our computational framework, and rigorous numerical analysis is provided to prove the convergence of our meshfree approximation approach. We also present several numerical experiments to validate the theoretical results of our approach and demonstrate the performance meshfree approximation in solving stochastic optimal control problems.

AMS subject classifications: 93E20, 65K10

Key words: Stochastic optimal control, maximum principle, backward stochastic differential equations, meshfree approximation.

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1 Introduction

The stochastic optimal control problem is a very important research topic in both mathematical and engineering communities. There is a large number of literatures contributing theoretical foundations to the optimal control theories [8,9,16,32], while others are presented with more emphasis on applications [11,25,34,35].

*Corresponding author. *Email address:* `bao@math.fsu.edu` (F. Bao)

Recently, the control theory found its new application in machine learning [1, 7], which reveals even broader application scenarios for optimal control.

In most practically applications, finding an optimal control with closed form is difficult–expect for some limited cases such as the linear-quadratic control problem, and people often need to obtain numerical solutions. There are typically two types of numerical methods to solve the stochastic optimal control problem: the dynamic programming and the stochastic maximum principle (SMP) [24]. The dynamic programming approach aims to transfer the control problem to numerical solutions for a class of nonlinear PDEs (Hamilton-Jacobi-Bellman, i.e. HJB equations), whose solutions can be interpreted in the viscosity sense [16, 25, 32]. There are several successful numerical schemes designed to solve the HJB equation numerically [5, 6, 13, 18]. The SMP approach, on the other hand, introduces an optimality condition for the optimal control. Then, the optimal control can be determined by an optimization procedure. In order to solve the stochastic optimal control problem through SMP, a system of backward stochastic differential equations (BSDEs) is derived as the adjoint process of the controlled state, and hence obtaining numerical solutions for BSDEs is required.

In this paper, we focus on the SMP approach due to its advantages over the dynamic programming approach in two-folds: (i) SMP allows to have some state constraints; (ii) SMP allows to have random coefficients in the state equation and in the cost functional. The general computational framework that we adopt is the gradient projection method [17], in which numerical schemes for BSDEs are used to calculate the gradient process of the cost functional, and the gradient projection optimization is applied to determine the optimal control [11, 24].

A major challenging in the existing gradient projection method for solving the stochastic optimal control problem lies in the spatial dimension approximation, which occurs in approximating conditional expectations for solutions of BSDEs. In low dimensional state spaces, one may compute values of conditional expectation at tensor-product grid points, and then use polynomial interpolation to approximate the entire conditional expectation. However, it is well-known that tensor-product grid points with polynomial interpolation suffer from the curse of dimensionality, hence the computational cost increases exponentially as the dimension increases. To address the curse of dimensionality, and to improve the efficiency of the current gradient projection method, in this work we introduce meshfree approximation methods to implement spatial dimension approximation.

The meshfree approximation methods typically avoid structured mesh grid points, which are required for polynomial interpolation. Therefore, meshfree methods are more flexible in embedding function features in the approximation