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Asymptotic Distribution of a Kind of Dirichlet Distribution*

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Abstract: The Dirichlet distribution that we are concerned with in this paper is very special, in which all parameters are different from each other. We prove that the asymptotic distribution of this kind of Dirichlet distributions is a normal distribution by using the central limit theorem and Slutsky theorem.

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1 Introduction

The Dirichlet distribution is a common multivariate distribution. It not only plays a very important role in modern non-parameter statistics, but also serves as the conjugate priori distribution of the multinomial distribution which is wildly used in Bayes statistics. As we know, the limit distribution of the Beta distribution is a normal distribution (see [1]), and the Dirichlet distribution is the multivariate form of the Beta distribution (see [2]). So it is very important to investigate the asymptotic distribution of the Dirichlet distribution. For the Dirichlet distribution in which all parameters are the same, we have obtained an important conclusion (see [3]). But the Dirichlet distribution that we are concerned with in this paper is very special, in which all parameters are different from each other. We prove that the asymptotic distribution of this kind of Dirichlet distributions is still a normal distribution by using the central limit theorem and Slutsky theorem.

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2 Main Result

Lemma 2.1[4] *Suppose that* $\xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ *are all i.i.d random vectors of dimension* $m(\in Z^+)$, $\bar{\xi}_n = \frac{1}{n}\sum\limits_{i=1}^{m}\xi_i$, *and* $N(x,\mu,\Sigma)$ *denotes the value on* x *of the multivariate normal distribution function with expectation* μ *and covariance matrix* Σ. *For* $E(\xi_j)=\mu$ *and* $Var(\xi_j)=\Sigma>0$, *there is* (*vector form* $\boldsymbol{a}<\boldsymbol{b}$ *expresses each of their corresponding components satisfies the same inequality relation*)

$$\lim_{n\to\infty} P\{\sqrt{n}(\bar{\xi}_n-\mu)<x\} \xrightarrow{W} N(x,0,\Sigma).$$

Lemma 2.2[5] *If* $\{\xi_n\}$ *and* $\{\eta_n\}$ *are all random variable sequences with* $(\xi_n,\ \eta_n)^T \xrightarrow{L} (\xi,\ \eta)^T$, $\zeta_n \xrightarrow{P} 0$ *and* $\tau_n \xrightarrow{P} 0$, *then* $(\xi_n+\zeta_n,\ \eta_n+\tau_n)^T \xrightarrow{L} (\xi,\ \eta)^T$.

Theorem 2.1 *Suppose that the random vector* $(Y_1,\ Y_2,\ \cdots,\ Y_n)^T$ *obeys the Dirichlet distribution with parameters* $(\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_m,\ \alpha_{m+1})$ *satisfying*

$$Y_i \geq 0, \quad \sum_{i=1}^{m} Y_i \leq 1, \quad \alpha_i \geq 1 \qquad (i=1,2,\cdots,m+1).$$

Particularly, when $\alpha_1=n_1,\ \alpha_2=n_2,\ \cdots,\ \alpha_m=n_m,\ \alpha_{m+1}=n_{m+1}$ $(n_i\in Z^+)$, *we denote* $\sum\limits_{k=1}^{m+1} n_k = N$. *Given a matrix* $\boldsymbol{A}$ *and a vector* $\boldsymbol{C}$, *if* $\frac{n_k}{N} \longrightarrow \gamma_k$ *when* $n_k\to\infty$, *where* $\gamma_k\in(0,1)$, $k=1,2,\cdots,m+1$, *then*

$$\boldsymbol{A}[(Y_1,\ Y_2,\ \cdots,\ Y_m)^T - \boldsymbol{C}] \xrightarrow{L} \boldsymbol{Z} \sim \boldsymbol{N}_m(\boldsymbol{0},\ \Sigma),$$

where

$$\boldsymbol{A} = (\lambda_{ij})_{m\times m}$$

with

$$\lambda_{ij} = \begin{cases} \dfrac{N^2}{\sqrt{n_i(N-n_i)N}}, & i=j; \\ 0, & i\neq j, \end{cases} \qquad i,j=1,2,\cdots,m,$$

and

$$\boldsymbol{C} = \left(\frac{n_1}{N},\ \frac{n_2}{N},\ \cdots,\ \frac{n_i}{N},\ \cdots,\ \frac{n_{m-1}}{N},\ \frac{n_m}{N}\right)^T,$$

and $\boldsymbol{N}_m(\boldsymbol{0},\Sigma)$ *is the* m*-dimensional normal distribution whose covariance matrix is*

$$\Sigma = (\sigma_{ij})_{m\times m} = \begin{cases} 1, & i=j; \\ -\sqrt{\dfrac{\gamma_i\gamma_j}{(1-\gamma_i)(1-\gamma_j)}}, & i\neq j, \end{cases} \qquad i,j=1,2,\cdots,m.$$

Proof. (1) Suppose that $X_{11}, X_{12}, \cdots, X_{1n_1}, \cdots; X_{21}, X_{22}, \cdots, X_{2n_2}, \cdots; \cdots; X_{m1}, X_{m2}, \cdots, X_{mn_m}, \cdots; X_{m+1,1}, X_{m+1,2}, \cdots, X_{m+1,n_{m+1}}, \cdots$ are all random variable sequences with independent and identical distribution and obey the exponential type distribution: $Exp(1)$. Because $Exp(1)$ is also $Ga(1,1)$ (see [6]), according to the countable additivity of Γ-distribution, we know that $n_i\bar{X}_i \sim \Gamma(n_i,1)$, where

$$\bar{X}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{ij}.$$