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On Weighted Possibilistic Mean, Variance and Correlation of Interval-valued Fuzzy Numbers*

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Abstract: In this paper, the concept of weighted possibilistic mean of interval-valued fuzzy number is first introduced. Further, the notions of weighted possibilistic variance, covariance and correlation of interval-valued fuzzy numbers are presented. Meantime, some important properties of them and relationships between them are studied.

Key words: Interval-valued fuzzy number, weighted possibilistic mean, weighted possibilistic variance, weighted possibilistic correlation

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1 Introduction

In 1987, Dubois and Prade[1] proposed an interval-valued expectation of fuzzy numbers, viewing them as consonant random sets. And later on, Carlsson[2] defined an interval-valued possibilistic mean of fuzzy numbers. Later, Fuller[3] also introduced a weighted possibilistic mean and variance of fuzzy numbers. The correlation of fuzzy numbers was also investigated by Carlsson[4]. The interval-valued fuzzy number (IvFN), as a generalization of ordinary fuzzy number, was introduced by Wang[5]. Recently, some important properties of interval-valued fuzzy numbers have been studied by Hong[6]. Moreover, interval-valued fuzzy numbers have been more widely applied to many fields (see [5], [7] and [8]), like fuzzy control, approximate reasoning, decision making and pattern recognition. Although some important properties of fuzzy numbers have been studied by many researchers (see [9]–[11]), those are not applicable for interval-valued fuzzy numbers. Therefore, in this paper, we briefly present and discuss the notions of the weighted possibilistic mean, variance, covariance and correlation of interval-valued fuzzy numbers by means of a weighted function which can measure the important degree of $\bar{\lambda}$-level set of interval-valued fuzzy numbers. These new

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notions proposed in this paper are consistent with the well-known definitions of expectation, variance, covariance and correlation in conventional theory of probability and mathematical statistics. All the theory developed in this paper is fully motivated by the principles and works introduced in the literature [2], [3] and [4].

The rest of this paper is organized as follows. In Section 2, we recall some basic concepts of interval-valued fuzzy number and demonstrate some of operations between interval-valued fuzzy numbers. Furthermore, the weighted interval-valued possibilistic mean and weighted possibilistic mean of interval-valued fuzzy numbers are introduced in Section 3. Finally, the notions of weighted possibilistic variance, covariance and correlation coefficient of interval-valued fuzzy numbers are proposed and discussed in Section 4.

2 Interval-valued Fuzzy Number

As a generalization of ordinary fuzzy number, the notion of interval-valued fuzzy number was suggested by Wang[5]. In the sequel, let

$$[I] = \{[a, b] \mid 0 \leq a \leq b \leq 1\}$$

be the family of all closed subintervals of [0,1].

Definition 2.1[12] *For any* $[a_1, b_1]$, $[a_2, b_2] \in [I]$, *we define a partial order relation* $\leq$ *in* $[I]$ *as*

$$[a_1, b_1] \leq [a_2, b_2] \qquad \textit{iff } a_1 \leq a_2 \textit{ and } b_1 \leq b_2.$$

Definition 2.2 *Let* $[a_1, b_1]$, $[a_2, b_2]$ *be any two closed intervals, some basic operations between them are defined as follows:*

$$[a_1, b_1] \vee [a_2, b_2] = [a_1 \vee a_2,\ b_1 \vee b_2],$$
$$[a_1, b_1] \wedge [a_2, b_2] = [a_1 \wedge a_2,\ b_1 \wedge b_2],$$
$$[a_1, b_1] + [a_2, b_2] = [a_1 + a_2,\ b_1 + b_2],$$
$$[a_1, b_1] - [a_2, b_2] = [a_1 - b_2,\ b_1 - a_2],$$
$$k[a_1, b_1] = \begin{cases} [ka_1, kb_1], & k \geq 0; \\ [kb_1, ka_1], & k < 0. \end{cases}$$

Definition 2.3 *A function* $A:\ \mathbf{R} \to [I]$, *where* $A(x) = [A^-(x),\ A^+(x)]$ *for any* $x \in \mathbf{R}$, *is named an interval-valued fuzzy number on* $\mathbf{R}$ *if the following conditions are satisfied*:

(1) *A is normal, i.e., there exists* $x_0 \in \mathbf{R}$, *such that* $A(x_0) = [1, 1]$;

(2) *For any* $\bar{\lambda} = [\lambda_1, \lambda_2] \in [I]/[0, 0]$, $A_{[\lambda_1, \lambda_2]}$ *is a closed bounded interval, where*

$$A_{[\lambda_1, \lambda_2]} = \{x \in \mathbf{R} \mid A^-(x) \geq \lambda_1 \textit{ and } A^+(x) \geq \lambda_2\} = [A_L([\lambda_1, \lambda_2]),\ A_U([\lambda_1, \lambda_2])].$$

Throughout this paper, we denote by $IvF^*(\mathbf{R})$ the family of all the interval-valued fuzzy numbers in $\mathbf{R}$, $\mathbf{R}$ represents the set of all real numbers.

Definition 2.4 *Let A be an interval-valued fuzzy number* (*see* [13]). *Then*