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Some Remarks on Distributional Chaos for Linear Operators*

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Abstract: In this paper, we consider some properties for bounded linear operators concerning distributional chaos. Norm-unimodality of bounded linear operators implies distributional chaos. Some properties such as similarity and spectra description for norm-unimodal operators are considered. The existence of distributional chaos in nest algebra is also proved. In addition, we obtain a sufficient and necessary condition of distributional chaos for a class of operators, which contains unilateral backward weighted shift operators.

Key words: distributional chaos, norm-unimodality, similarity, spectrum, nest algebra

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1 Introduction

A discrete dynamical system is simply a continuous function $f: X \to X$, where X is a complete separable metric space. As we know, the main task to investigate the system $(X,\ f)$ is to clear how the points of X move, i.e., to clear the "orbits". For $x \in X$, the orbit of x under f is

$$\mathrm{Orb}(f, x) = \{x,\ f(x),\ f^2(x),\ \cdots\},$$

where $f^n = f \circ f \circ \cdots \circ f$ is the n-th iterate of f obtained by composing f with n times.

We are interested in the dynamical systems induced by continuous linear operators on Banach spaces. To see the progress of this aspect, we refer the reader to [1] and [2]. In this article, we restrict our attention to distributional chaos for continuous linear operators. In [3], distributional chaos is defined in the following way.

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For any pair $\{x, y\} \subset X$ and any $n \in \mathbf{N}$, define distributional function $F^n_{xy}: \mathbf{R} \to [0, 1]$ by

$$F^n_{xy}(\tau) = \frac{1}{n}\#\{0 \le i \le n-1;\ d(f^i(x),\ f^i(y)) < \tau\}.$$

Furthermore, define

$$F_{xy}(\tau) = \liminf_{n\to\infty} F^n_{xy}(\tau), \qquad F^*_{xy}(\tau) = \limsup_{n\to\infty} F^n_{xy}(\tau).$$

Both F_{xy} and F^*_{xy} are nondecreasing functions and may be viewed as cumulative probability distributional functions satisfying

$$F_{xy}(\tau) = F^*_{xy}(\tau) = 0 \qquad \text{for } \tau < 0.$$

Definition 1.1 *$\{x, y\} \subset X$ is said to be a distributionally chaotic pair, if for any $\tau > 0$,*

$$F^*_{xy}(\tau) \equiv 1,$$

and there exists $\epsilon > 0$ such that $F_{xy}(\epsilon) = 0$; f is called distributionally chaotic, if there exists an uncountable subset $D \subseteq X$ such that each pair of two distinct points is a distributionally chaotic pair, where D is called a distributionally ϵ-scrambled set.

Martínez-Giménez *et al.*[4] gave a sufficient condition to distributional chaos for shift operators. Hou *et al.*[5] introduced a property called norm-unimodal which implies distributionally chaotic.

Definition 1.2 *Let X be a Banach space and $T \in \mathcal{L}(X)$. T is called norm-unimodal, if there exists a constant $\gamma > 1$ such that for any $m \in \mathbf{N}$, there exists $x_m \in X$ satisfying*

$$\lim_{k\to\infty} \|T^k x_m\| = 0$$

and

$$\|T^i x_m\| \ge \gamma^i \|x_m\|, \qquad i = 1, 2, \cdots, m.$$

Furthermore, such γ is said to be a norm-unimodal constant for the norm-unimodal operator T.

Theorem 1.1 (Distributionally Chaotic Criterion) *Let X be a Banach space and $T \in \mathcal{L}(X)$. If T is norm-unimodal, then T is distributionally chaotic.*

However, the converse implication is not true in general. In [6], one can see that norm-unimodal operators are dense in the set of distributionally chaotic operators in the sense of norm topology. Moreover, the interior of the set of distributionally chaotic operators is contained in the set of norm-unimodal operators. These together show that, in most cases, norm-unimodality can be used as a criterion when we investigate whether an operator is distributionally chaotic.

In this article, we continue to discuss some properties such as similarity and spectra description for norm-unimodal operators. Moreover, the existence of distributional chaos in nest algebra is proved. We also obtain a sufficient and necessary condition of distributional chaos for a class of operators, which contain unilateral backward weighed shift operators.