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Strong Converse Inequality for the Meyer-König and Zeller-Durrmeyer Operators*

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Abstract: In this paper we give a strong converse inequality of type B in terms of unified K-functional $K_\lambda^\alpha(f, t^2)$ $(0 \le \lambda \le 1,\ 0 < \alpha < 2)$ for the Meyer-König and Zeller-Durrmeyer type operators.

Key words: Meyer-König and Zeller-Durrmeyer type operator, moduli of smoothness, K-functional, strong converse inequality, Hölder's inequality

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1 Introduction

The Meyer-König and Zeller operators were given by

$$\begin{aligned} M_n(f, x) &= \sum_{k=0}^{\infty} f\Big(\frac{k}{n+k}\Big) m_{n,k}(x), \qquad 0 \le x < 1, \\ M_n(f, 1) &= f(1), \\ m_{n,k}(x) &= \binom{n+k}{k} x^k (1-x)^{n+1}, \end{aligned}$$

which were the object of several investigations in approximation theory (see [1–3]). In recent years there are many results of strong converse inequalities for various operators (see [4–7]). Since the expression of the moment of the Meyer-König and Zeller type operators is very complicated (see [8–10]), we have not seen any result of strong converse inequality for Meyer-König and Zeller-Durrmeyer type operators. In this paper, we study the modification of Meyer-König and Zeller-Durrmeyer type operators $\tilde{M}_n(f, x)$:

$$\tilde{M}_n(f, x) = \sum_{k=0}^{\infty} \Phi_{n,k}(f) m_{n,k}(x), \qquad f \in C[0, 1],$$

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where

$$\begin{aligned}
&\Phi_{n,k}(f)=C_{n-2,k-1}^{-1}\int_0^1 f(t)m_{n-2,k-1}(t)\mathrm{d}t,\\
&m_{n,k}(x)=\binom{n+k}{k}x^k(1-x)^{n+1},\\
&m_{n,-1}(x):=0,\\
&C_{n,k}=\int_0^1 m_{n,k}(t)\mathrm{d}t=\frac{n+1}{(n+k+1)(n+k+2)},
\end{aligned}$$

and give a strong converse inequality of type B.

We recall that for $0\le\lambda\le 1$, and $\varphi(x)=\sqrt{x}(1-x)$,

$$\omega_{\varphi^\lambda}^2(f,t)=\sup_{0<h\le t}\|\Delta_{h\varphi^\lambda}^2\|,$$

where

$$\|f\|:=\sup_{x\in[0,1)}|f(x)|,$$

$$\Delta_{h\varphi^\lambda}^2 f(x)=\begin{cases} f(x+h\varphi^\lambda(x))-2f(x)+f(x-h\varphi^\lambda(x)), & \text{if } x\pm h\varphi^\lambda(x)\in[0,1);\\ 0, & \text{otherwise,}\end{cases}$$

and

$$K_{\varphi^\lambda}^2(f,t^2)=\inf_{g\in D}\{\|f-g\| \ +t^2\|\varphi^{2\lambda}g''\|\},$$

where

$$D=\{g \mid g'\in A.C._{\mathrm{loc}}, \|\varphi^{2\lambda}g''\|<\infty\}.$$

In this paper we use the relation $\omega_{\varphi^\lambda}^2(f,t)\sim K_{\varphi^\lambda}^2(f,t^2)$ (see [11]), which means that, there exists a positive constant C such that

$$C^{-1}K_{\varphi^\lambda}^2(f,t^2)\le\omega_{\varphi^\lambda}^2(f,t)\le CK_{\varphi^\lambda}^2(f,t^2).$$

Before state our results, we give some new notations.

For $0\le\lambda\le 1$, $0<\alpha<2$, and $\varphi(x)=\sqrt{x}(1-x)$,

$$\begin{aligned}
&C_0=\{f\in C[0,1],\ f(0)=f(1)=0\}, && \|f\|_0=\sup_{x\in(0,1)}|\varphi^{\alpha(\lambda-1)}(x)f(x)|,\\
&C_{\lambda,\alpha}^0=\{f\in C_0,\ \|f\|_0<\infty\}, && \|f\|_2=\sup_{x\in(0,1)}|\varphi^{2+\alpha(\lambda-1)}(x)f''(x)|,\\
&C_{\lambda,\alpha}^2=\{f\in C_0,\ \|f\|_2<\infty, f'\in A.C._{\mathrm{loc}}\},\\
&K_\lambda^\alpha(f,t^2)=\inf_{g\in C_{\lambda,\alpha}^2}\{\|f-g\|_0+t^2\|g\|_2\}, \qquad f\in C_0.
\end{aligned}$$

The main results of this paper can be stated as follows.

Theorem 1.1 *Suppose* $0\le\lambda\le 1$, $0<\alpha<2$, *and* $f\in C_{\lambda,\alpha}^0$. *Then there exists a constant* $K>1$ *such that for* $l\ge Kn$ *we have*

$$K_\lambda^\alpha\Big(f,\frac{1}{n}\Big)\le C\frac{l}{n}(\|\tilde{M}_n f-f\|_0+\|\tilde{M}_l f-f\|_0).$$

Throughout this paper, C denotes a positive constant independent of n and x, which are not necessarily the same at each occurrence.