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The Study of Electromagnetic Scattering by a Non-perfectly Conductor in Chiral Media by Potential Theory*

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Abstract: This paper is concerned with the electromagnetic scattering by a non-perfectly conductor obstacle in chiral environment. A two-dimensional mathematical model is established. The existence and uniqueness of the problem are discussed by potential theory.

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1 Introduction

Chiral materials have been studied intensively in electromagnetic theory literature in recent years. Chiral media are examples of media responding with both electric and magnetic polarization to either electric or magnetic excitation. Chiral media can be characterized by a set of constitutive relations in which the electric and magnetic fields are coupled.

The electromagnetic scattering by a perfectly conductor in chiral environment has been studied in [1]. In this paper we consider that the conductor is no longer perfectly. The main difference with the problem in [1] is that the boundary condition is changed to the impedance boundary condition. The two-dimensional model is established, and we studied the model by the potential theory.

We start our consideration with time-harmonic Maxwell's equations:

$$\nabla \times \boldsymbol{E} - \frac{\mathrm{i}k}{\mu}\boldsymbol{B} = 0, \qquad \nabla \times \boldsymbol{H} + \frac{\mathrm{i}k}{\varepsilon}\boldsymbol{D} = 0,$$

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where $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{D}$ and $\boldsymbol{B}$ denote the electric field, magnetic field, electric and magnetic displacement vectors in $\mathbf{R}^3$, respectively, and ε is the electric permittivity, μ is the magnetic permeability.

In a homogeneous isotropic chiral medium the following Drude-Born-Fedorov constitutive equations hold:

$$\boldsymbol{D} = \varepsilon(\boldsymbol{E} + \beta\nabla \times \boldsymbol{E}), \qquad \boldsymbol{B} = \mu(\boldsymbol{H} + \beta\nabla \times \boldsymbol{H}),$$

where β is the chirality admittance.

Eliminating $\boldsymbol{B}$ and $\boldsymbol{D}$, we get

$$\nabla \times \boldsymbol{E} = \gamma^2\beta\boldsymbol{E} + \mathrm{i}\frac{\gamma^2}{k}\boldsymbol{H}, \tag{1.1}$$

$$\nabla \times \boldsymbol{H} = \gamma^2\beta\boldsymbol{H} - \mathrm{i}\frac{\gamma^2}{k}\boldsymbol{E}, \tag{1.2}$$

where $k = \omega\sqrt{\varepsilon\mu}$, $\gamma^2 = \dfrac{k^2}{1 - k^2\beta^2}$. With respect to the physical parameters, we make the additional assumption that $\varepsilon > 0$, $\mu > 0$, $\beta > 0$ and $|k\beta| < 1$.

In chiral medium, left-handed and right-handed fields can both propagate independently and with different phase speeds (see [2]). So we consider the decomposition of $\boldsymbol{E}$, $\boldsymbol{H}$ into suitable Beltrami fields $\boldsymbol{Q}_L$, $\boldsymbol{Q}_R$ (see [3]) as

$$\boldsymbol{E} = \boldsymbol{Q}_L + \boldsymbol{Q}_R, \qquad \boldsymbol{H} = -\mathrm{i}(\boldsymbol{Q}_L - \boldsymbol{Q}_R).$$

Then together with (1.1) and (1.2), we obtain that

$$\nabla \times \boldsymbol{Q}_L = \gamma_L\boldsymbol{Q}_L, \qquad \nabla \times \boldsymbol{Q}_R = -\gamma_R\boldsymbol{Q}_R, \tag{1.3}$$

where the wave numbers γ_L, γ_R of $\boldsymbol{Q}_L$, $\boldsymbol{Q}_R$, respectively, are given as follows:

$$\gamma_L = \frac{k}{1 - k\beta}, \qquad \gamma_R = \frac{k}{1 + k\beta}.$$

Assume that $\tilde{D} = \{\boldsymbol{x} \in \mathbf{R}^3 \mid (x_1, x_2) \in D,\ x_3 \in \mathbf{R}\}$ is an infinitely long cylinder parallel to x_3-direction, and $\partial\widetilde{D}$ is the surface of $\widetilde{D}$. Here D is the cross-section of $\tilde{D}$ in x_1-x_2-plane. We assume that D is simply connected with $C^{2,\alpha}$-boundary ∂D, where $\alpha \in (0, 1)$.

We assume that the incident waves in chiral medium are plane waves, so the incident waves also have the decomposition

$$\boldsymbol{E}^i = \boldsymbol{Q}_L^i + \boldsymbol{Q}_R^i = \boldsymbol{q}_L\mathrm{e}^{\mathrm{i}\gamma_L\boldsymbol{p}_L\cdot\boldsymbol{x}} + \boldsymbol{q}_R\mathrm{e}^{\mathrm{i}\gamma_R\boldsymbol{p}_R\cdot\boldsymbol{x}}, \tag{1.4}$$

$$\boldsymbol{H}^i = -\mathrm{i}(\boldsymbol{Q}_L^i - \boldsymbol{Q}_R^i) = -\mathrm{i}(\boldsymbol{q}_L\mathrm{e}^{\mathrm{i}\gamma_L\boldsymbol{p}_L\cdot\boldsymbol{x}} - \boldsymbol{q}_R\mathrm{e}^{\mathrm{i}\gamma_R\boldsymbol{p}_R\cdot\boldsymbol{x}}), \tag{1.5}$$

where $\boldsymbol{q}_L$, $\boldsymbol{q}_R$ are the polarization vectors, and $\boldsymbol{p}_L$, $\boldsymbol{p}_R$ the propagation unit vectors.

Note that $\boldsymbol{Q}_L^i$ and $\boldsymbol{Q}_R^i$ satisfy (1.3) and $\nabla \cdot \boldsymbol{E}^i = 0,\ \nabla \cdot \boldsymbol{H}^i = 0$, so we have

$$\boldsymbol{p}_L \cdot \boldsymbol{q}_L = 0, \qquad \boldsymbol{p}_L \times \boldsymbol{q}_L = -\mathrm{i}\boldsymbol{q}_L, \qquad \boldsymbol{p}_R \cdot \boldsymbol{q}_R = 0, \qquad \boldsymbol{p}_R \times \boldsymbol{q}_R = \mathrm{i}\boldsymbol{q}_R. \tag{1.6}$$

Let a plane wave be incident on a non-perfectly conducting obstacle $\tilde{D}$ in chiral medium. So we have the boundary condition

$$\widetilde{\nu} \times (\nabla \times \boldsymbol{E}) - \mathrm{i}\lambda(\widetilde{\nu} \times \boldsymbol{E}) \times \widetilde{\nu} = 0 \qquad \text{on } \partial\widetilde{D}, \tag{1.7}$$

where λ is the impedance coefficient of non-perfectly conducting obstacle.

And the radiation conditions are

$$\begin{aligned} &\frac{\boldsymbol{x}}{|\boldsymbol{x}|} \times \boldsymbol{H}^s + \boldsymbol{E}^s = o\Big(\frac{1}{|\boldsymbol{x}|}\Big), \qquad |\boldsymbol{x}| \to \infty, \\ &\frac{\boldsymbol{x}}{|\boldsymbol{x}|} \times \boldsymbol{E}^s - \boldsymbol{H}^s = o\Big(\frac{1}{|\boldsymbol{x}|}\Big), \qquad |\boldsymbol{x}| \to \infty \end{aligned} \tag{1.8}$$